Competitive Capture of Public Opinion*

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Abstract

We propose a general equilibrium model where two special interest groups (SIGs) compete to influence public opinion. Citizens with heterogeneous priors over a binary state of the world receive reports drawn from a continuous message space by a variety of sources. The two opposite SIGs attempt to push their own agenda (one SIG to persuade citizens towards one state of the world, the other towards the alternative state of the world) by capturing the messages these sources convey. We characterize the equilibrium level of capture of each source by competing SIGs as well as the equilibrium level of information transmission. We show that capture increases the prevalence of the *ex ante* most informative messages. As a consequence, rational citizens discount such informative reports. Opposite capturing efforts do not cancel each other and result in a loss of social learning. We show that efforts to capture an information source are strategic substitutes: citizens' skepticism of messages favoring the view of the SIG that is expected to capture that source dampen the incentives of the opposite SIG. Strategic substitution exacerbates horizontal differentiation so the information landscape becomes more polarized. We finally show that increased demand for information when SIGs want to fire up the base can exacerbate differentiation, increase capture, and reduce information transmission in equilibrium.

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1 Introduction

Since public opinion over issues shapes which policies can be implemented, special interest groups (henceforth SIGs) care about beliefs in the population. One method of shaping public opinion is to influence the sources of information that reach citizens so that issues of interest are covered in a favorable way. Traditional media are often subject to influence which affects its coverage, and SIGs exert this pressure using a variety of ways which range from leveraging economic relationships such as advertising to outright ownership.[1] However, efforts to shape the information that reaches the public are not limited to traditional media. For example, Oreskes and Conway (2010) describe how scientists deeply connected to conservative funding sources have inserted themselves in the scientific debate to cast doubt on the scientific consensus over issues ranging from the harmful effects of smoking to global warming.[2] Posner (2008) and Conley and Ruy (2022) show that religious leaders have been deployed to disseminate a worldview favorable to the interests of national and international SIGs.[3] SIG are also using concerted campaigns through social media to influence public opinion.[4]

These examples suggest that SIGs channel their influence through a variety of information sources with various degrees of credibility and which reach different segments of the public. Moreover, for many policy domains – ranging from climate policies to reproductive rights – groups are organized on opposite sides of an issue and are therefore competing over public opinion. Crucially, while SIGs care about the beliefs and attitudes of the public, they cannot directly manipulate

[1] For example, Petrova (2008) describes how a SIG successfully lobbied media to spread the use of the term "death tax" to refer to the inheritance tax. Beattie, Durante, Knight, and Sen (2021) shows how newspaper coverage of car recalls varies as a function of car advertisement revenue, and Durante, Fabiani, Laeven, and Peydro (2021) shows that media-bank links colored media coverage of the European debt crisis. Martin and McCrain (2019) shows that ownership by the Sinclair Broadcast Group changes the content of news report from TV channels.

[2] Shapiro (2016) offers a rich analysis of the interaction between journalists and SIG in the context of climate change.

[3] Posner (2008) probes the links between evangelist preachers and partisan interests in the USA. Conley and Ruy (2022) describes Putin's deployment of multiple channels of influence in the West: "Through media, NGOs, political parties, Russian officials, and norms entrepreneurs [religious leaders], the Kremlin effectively challenges the tenets of Western liberalism. These channels spread the argument that liberalism threatens religious beliefs, which in turn threatens the national identity that is so closely tied to these beliefs."

[4] Conley, Mina, Stefanov, and Vladimirov (2016), Gosh and Scott (2018) among many others explore how social media is being actively exploited as disseminator of ideas by international and domestic interest groups. Allcott and Gentzkow (2017) examine the partisan content of social media in the run-up to the 2016 presidential elections in the USA.

them. They instead try to shape public opinion indirectly by capturing how information sources cover reality.[5] This implies that a proper analysis of these influence activities must take into account how citizens update their views – the object of lobby interest – when they suspect the coverage of an issue may be tainted by SIG influence.

These strategic interactions at multiple levels pose several questions. What type of coverage and information sources are favored by SIGs competing over public opinion? How does the "court of public opinion" react to the presence of competing SIGs? Do SIGs with opposite interests cancel each other in their influencing activities? Do opposing SIGs focus on the same information sources or focus their pressure on different sources? How do the strategies of SIGs which try to fire up their base differ from those who try to moderate those who are opposed to their views?

To make headway on these questions, we propose a model with two SIGs, left and right, multiple (possibly heterogenous) information sources and citizens with heterogeneous priors over a binary state of the world. SIGs care about the posterior beliefs of the public and are diametrically opposed: one SIG wants citizens to update towards one state of the world and the other SIG wants them to update in the opposite direction. SIGs can simultaneously and covertly spend resources to capture how each source informs citizens about the world. In the absence of capture, each information source acts as a Blackwell experiment: it receives a continuous informative signal on the state of the world and proceeds to honestly convey the signal to the public it reaches. However, if capture is successful, the triumphant SIG can induce the captured source to convey *any* message in the continuous set. Citizens reached by the source observe the conveyed message and rationally update their beliefs.

Several features of this model are noteworthy and motivated by the questions we pose. We consider receivers with heterogeneous priors to capture the multiplicity of views present in public opinion. In allowing a continuous message space, we depart from filtering models in which the message space is restricted to be binary so we can analyze which kinds messages are emphasized by SIG. Third, receivers are uncertain about the motives of the sender. Finally, messages are not certifiable and there is no commitment to either the resources spent to capture information sources

[5] In contrast, the canonical political lobbying literature has focused on quid-pro-quo exchanges in which government, in exchange for SIG funds, delivers policy: the object that the lobby directly cares about. See, among others, Grossman and Helpman (2001).

or the communication strategy of SIGs.

We characterize the equilibrium strategies of SIGs as well as the equilibrium information transmission and obtain several important insights about competitive information manipulation. First, SIG capture leads to polarization in observed messages: messages with extreme likelihood ratios, which would be very informative in the absence of capture, become more frequent in the presence of capture. In contrast, centrist messages, which are less informative, are observed less often. This is because the optimal manipulation strategy of a SIG is to mix over a set of messages in such a way that the posterior of a citizen is equalized upon observing any message plausibly conveyed by a captured source. To build intuition, note that for each possible message, a rational citizen needs to weigh two possibilities. On the one hand, how informative that message would be if the source was honest (i.e. if it was not captured by an SIG). On the other hand, how likely it is that the source was captured and induced to send the observed message. Therefore, if an SIG's strategy is to always send the most favorable message, citizens would easily infer such a message was the result of capture and would disregard it. Facing this reaction, the best either SIG can do is to mix across a range of relatively favorable messages. In equilibrium, citizens treat each suspect message with educated skepticism, which means that polarization in expected messages does not necessarily imply polarization in citizens' expected posteriors.

Second, despite the fact that capture leads to the publication of more extreme messages –which would be more informative if taken at face value– there is less learning in equilibrium. This is because, as noted, the possibility of capture makes rational citizens skeptical of messages that are too favorable to either state. After all, these are the messages that SIGs are sending if they manage to capture the source. This phenomenon is extremely deleterious to social learning: the messages that would lead to faster updating about the true state of the world, are the ones that are being jammed by the SIGs and therefore rationally discounted by the public. It follows that competing SIGs do not cancel each other: they degrade the overall informativeness of the environment.

Third, due to the rational skepticism of citizens, capturing efforts by the two SIGs are strategic substitutes at each information source. The higher the effort exerted by, say, the left-SIG, the more skeptical citizens are when they observe messages favorable to the left state of the world.

This limits the leftward shift of citizens' beliefs reached by the source, and therefore reduces the marginal benefit of capture perceived by the right-SIG. This force amplifies source slant (expected coverage which is lopsided in favor of left or right, a result of SIGs exerting different levels of capturing effort) in equilibrium.

Fourth, we explore comparative statics of capturing effort with respect to beliefs. We first show that an "optimistic" SIG –i.e. a SIG which high priors that their preferred state of the world is true– have lower incentives to capture as they expect honest sources to be favorable. We then explore how beliefs in the audience affect incentives to capture. In order to do this, we note that SIGs may be interested in reaching different segments of the population. For example, if the goal is to incite a partisan riot, an SIG would like to reach those whose priors are aligned with the party and reinforce them. We call this "firing up the base." In contrast, if the purpose is to induce doubt in the opponents' camp, then the SIG would like to reach those with opposed priors and bring them towards the center, which we call "moderating the opposition." We show that in our framework, the curvature of SIG preferences captures these two motivations. If both SIGs share the same motivation, then a shift in audience priors to the right or the left necessarily leads one SIG to increase effort and the other to decrease it. This leads to an increase in expected slant if the shift favors the SIG that was exerting more effort.

These foundational results rely on rational updating by Bayesian citizens. This raises the concern that the findings may not be robust to the presence of unsophisticated agents among the public. To explore this possibility we consider an extension in which a fraction of the population is *naive* – citizens which ignore the possibility of source capture and always take the messages at face value. While this addition naturally increases returns to capture, we show that the main insights of the model survive: SIG activity increase the prevalence of extreme messages, rational citizens react with skepticism, social learning suffers, and capturing efforts are strategic substitutes. In other words, the presence of *naive* viewers, whose vulnerability to manipulation is very high, does not result in SIGs disregarding the share of public opinion which is sophisticated.

After the main results, we explore some of their consequences for an informational landscape comprising several sources. We present two results to illustrate how strategic substitution at the source level, together with general equilibrium considerations, exacerbates horizontal differentiation in slant. First, we consider a situation with two information sources and present comparative statics on polarization: we show that strategic substitution amplifies the expected difference in SIG strategies across sources, and thus the expected horizontal differentiation in slant. Second, we show that in an environment with n ex ante identical sources, a local asymmetry in just one source which favors one of the SIG (perhaps because it is exogenously cheaper for that SIG to exert influence on this particular source) spreads in equilibrium over the rest of the information landscape creating horizontal differentiation in slant for *all* sources.

We then allow citizens to endogenously choose the information source that will be most useful to them. We show that this leads to partial sorting: under general conditions, citizens that have leftist priors will sort into sources most likely captured by the left SIG, and the same is true at the other end of the distribution of priors. Centrists, however, may sort non-monotonically. Interestingly, audience sorting across sources does not necessarily mean more horizontal differentiation in slant. It only exacerbates source polarization when firing up the base is the main concern of SIGs. Finally, we also show that, perhaps paradoxically, higher demand for unbiased information may lead to a less informative source landscape. Higher demand for information increases rational sorting by citizens into aligned information sources. When SIGs want to fire up their base, the resulting increase in capturing incentives may overwhelm the informational benefits of sorting.

These results show that our model supports equilibria with several empirically appealing features. First, coverage of an issue will likely be systematically different across sources, with some sources aligning with the left and others with the right. Second, rational citizens largely sort according to their priors, but nonetheless are skeptical of the information they consume. In this sense, despite the lies, consumers are not systematically deceived. This is consistent with recent evidence that while viewers seek information sources with which they are ideologically aligned, they often question the veracity of information and do not update according to the news' literal meaning.[6]

[6] Gentzkow and Shapiro (2010) show robust alignment between a media outlet's slant and their viewership. Angelucci and Prat (2021) find that most viewers are able to identify fake political news. Martin and Yurukoglu (2017) find that cable news have progressively polarized in terms of coverage but that ideological polarization in the population is proportionally much smaller, which is in line with existing research in political science (Ansolabehere,

We contribute to the theoretical literature on the political economy of media capture. This literature has advanced dramatically in recent decades.[7] The incumbent government is the primordial example of a capturing agent, as shown in most detail in McMillan and Zoido (2004). Models of government capture of media focus therefore on the case with a single special interest group. Besley and Prat (2006) relies on a disclosure game where printed news are never lies.[8] In Gehlbach and Sonin (2014) commitment to an editorial line means media filter information, but do not distort it.[9] Similarly, Petrova (2008) focuses on capture by a single social group –the rich– and assumes exogenous costs of lying by the media. Corneo (2006) and Shapiro (2016), in contrast, offer models with multiple SIG potentially capturing a single media outlet. Prat (2018) considers multiple media platforms and characterizes robust upper bounds on the ability of a SIG to influence beliefs. These existing models consider viewers with homogeneous priors and limit the message space to a binary signal. We advance on the literature by considering SIGs with opposing interests, which influence multiple information sources which reach viewers with heterogeneous priors.[10] In addition, we put no restrictions on the message space and assume no commitment to a publishing rule. These features allow us to have predictions on i. differential capture across sources by the different SIGs; ii. the polarizing effects of capture on published news; and iii. the resulting compression of viewers' beliefs. We also characterize SIG attitudes towards audience segments by analyzing when SIG prioritize firing up the base as opposed to moderating the opposition.

The theoretical literature on media economics has also been preoccupied with the co-existence of outlets with different slants. Arguments have been offered for supply and demand drivers of such polarization.[11] We contribute to this literature by noting that influence efforts by SIGs

Rodden, and Snyder, 2006). In a recent experiment, Brookman and Kalla (2022) show that partisans forced to watch media with opposite slant moderately revise their views but do not fundamentally change their partisan affiliation or presidential vote, and return to their partisan media as soon as the experiment concludes. Gentzkow, Shapiro, and Sinkinson (2011) similarly find that the introduction of partisan newspapers did not affect party vote shares.

[7]For a theoretical survey see Prat (2015)

[8]See Milgrom (1981) Dye (1985) Milgrom and Roberts (1986) , and Okuno-Fujiwara, Postlewaite, and Suzumura (1990) for certifiable disclosure of private information.

[9]See Kamenica and Gentzkow (2011) for the analysis of information transmission when the sender can commit to the disclosure rule, and Bergemann and Morris (2019) for a survey of this class of models and their applications. Gitmez and Molavi (2022) also follows this modeling tradition and considers heterogeneous receivers but a single sender.

[10]To our knowledge, Petrova (2012) is the only previously existing model with multiple lobbyists and media outlets. However, it is not a model with information transmission.

[11]For a theoretical survey see Gentzkow, Shapiro, and Stone (2015). Three main drivers have been proposed.

are strategic substitutes. This force will exacerbate horizontal differentiation across sources, thus reinforcing any of the proposed main drivers of polarization. Relatedly, important contributions such as Suen (2004) and Gentzkow and Shapiro (2006) theoretically generate alignment between consumer ideology and media slant. They do so in the context of binary messages. In Suen (2004), where senders coarsen a continuous signal, a receiver's utility increases as the media slants coverage closer to the receiver's own prior. We instead obtain sorting of consumers into aligned media in a distortion model with continuous message space and no commitment. In our model, capture, which leads to slant, unambiguously reduces receivers' utility. However, receivers prefer to get news from aligned outlets because they get even less value from unaligned ones.[12]

We also contribute to the literature on strategic communication where the sender may have uncertain motives –including the possibility that he reports honestly– and to the literature where the receiver may be naïve or strategically unsophisticated. Sobel (1985) shows how a biased sender can maintain a reputation for honesty when an honest sender always tells the truth.[13] In our setup, the honest source also relays the truth to the public, although capturing SIGs do not have an incentive to build a reputation for honesty.[14] Morgan and Stoken (2003) and Li and Madarasz (2008) show that information transmission may be reduced if the sender discloses his preferences. In our model, however, knowing the identity of the source would always lead to (weakly) more informative media. Thus, in our setup concealment of motives reduces information transmission but incentivizes capture. Wolinsky (2003) and Dziuda (2011) study models with partial verifiability: the sender may be biased in favor or against a given issue, but can only conceal evidence, not fabricate them. Interestingly, their equilibria also feature receivers' skepticism towards extreme views and a constant posterior belief generated by extreme messages.[15] We obtain

First, suppliers such as owners or journalists may have different ideologies which they are trying to push on the population (i.a. Baron, 2006; Anderson and McLaren, 2012). Second, rational demand for news by viewers with heterogeneous priors or ideology can lead to a segmented market (i.a. Chan and Suen, 2008; Gentzkow and Shapiro, 2006; Sobbrio, 2014). Finally, demand effects may also be due to cognitive biases or other ideological effects on consumer demand (i.a. Gabszewicz, Laussel, and Sonnac, 2001; Mullainathan and Shleifer, 2005; Bernhardt, Krasa, and Polborn, 2008).

[12] The mechanism in our model is reminiscent of Cukierman and Tommasi (1998).

[13] See also Shin (1993) and Morris (2001).

[14] For communication with behavioral honest types, see Benabou and Laroque (1992), Chen (2011), and Kim and Pogach (2014).

[15] In Wolinsky (2003), the sender can underreport the state but never overreport it. In equilibrium, any message above a threshold is fully trusted, while messages below that threshold lead to the same posterior. This equilibrium

a similar equilibrium structure despite the fact that in our model SIGs are free to fabricate the news.

Finally, Glazer, Herrera, and Perry (2020) considers a biased sender that can costlessly misrepresent a fake review as honest, while Chen (2011) studies Crawford-Sobel's constant-bias leading example where the sender may be honest and the receiver may be naive – in which case she believes every message is sent by an honest sender.[16] While the communication equilibria in these papers share some of the same features of the communication equilibria in Sections 3 and 5 –most notably, sender exaggeration, receiver skepticism, and message clustering– our analysis allows for players with heterogeneous priors and our main focus is on endogenizing the levels of source capture –i.e., the probability that the sender remains honest– which is exogenously set in those papers.[17] In fact, certain simplifying features of our model of communication with prior heterogeneity allow us to solve for the general equilibrium model in which ex ante heterogeneous viewers sort across endogenously captured information sources.

The rest of the paper is organized as follow. Section 2 sets out the model. Section 3 describes the optimal lying strategy of SIGs and its effects on message distribution and information transmission. Section 4 studies incentives to capture a monopoly source of information, shows that capturing efforts are strategic substitutes and explores how the heterogeneous priors of citizens affect capturing incentives. Section 5 demonstrates that our main results do not hinge on assuming that the citizenry is fully sophisticated. Section 6 introduces multiple information sources and shows that the model supports horizontal differentiation and Section 7 explores the implications of audience sorting across information sources. We then offer some conclusions.

2 Model

We propose the following model in which endogenously manipulated information reaches the public. There are $n \geq 1$ possibly heterogeneous information sources which cover issues related to an

is similar to our model in which only the L-SIG engages in capture. In Dziuda (2011), the sender privately obtains several arguments in favor or against an issue and can conceal arguments. For the case of a single type of bias, equilibria exhibit, as in our model, receiver's skepticism when a small number of arguments either in favor or against are presented.

[16] See also Kartik, Ottaviani, and Squintani (2007)

[17] For persuasion with heterogeneous priors, see den Steen (2004), Che and Kartik (2009), and Alonso and Câmara (2016).

underlying binary state of the world. There are two SIGs with opposed preferences over citizens' beliefs on the state. For example, the underlying state of the world may be the gravity of the climate crisis and the information sources range from panels of experts assembled by international institutions to media channels with questionable objectivity. Carbon-dependent energy companies may want to downplay the weight of evidence linking current weather events with global warming, while climate activists may want to highlight it. These SIGs can covertly devote resources to capture the informative coverage about the state of the world, separately for each information source. Citizens receive a message from an information source and discount it according to the anticipated level of capture.

State space and Prior Beliefs: There is an unknown state $\theta \in \Theta = \{-1, 1\}$. Citizens have heterogeneous prior beliefs $p = \Pr[\theta = 1]$ over the state, with a mass $F_p(p)$ of citizens with priors not exceeding p and $M = \int_0^1 dF_p(p)$ the total mass of citizens.

Special Interest Groups and Information Sources: There are two strategic SIGs, R and L, with opposed preferences. Specifically, R wants to induce in citizens the highest posterior belief over θ while L wants to induce the lowest. If μ is the posterior belief of a citizen, then the SIGs utility functions are $v_R(\mu)$ and $v_L(\mu)$ with v_R strictly increasing and v_L strictly decreasing with $|v_i'|$, $i \in \{L, R\}$, bounded away from zero. Thus, if $\mu(m; p)$ is the posterior belief of a citizen with prior p after observing message m, then the indirect utility over messages of i- SIG, $i \in \{R, L\}$, facing a public characterized by $F_p(p)$, is

$$V_i(m) \equiv \int_0^1 v_i\left(\mu(m; p)\right) dF_p(p). \tag{1}$$

There are $n \geq 1$ different information sources, whose coverage of an issue can be captured by a SIG. Sources function as a Blackwell-experiment: they observe an informative signal $m^j \in \mathscr{M} \subset \mathbb{R}$, with $j \in \{1, ...n\}$ indexing sources, which is generated according to $\Pr[m^j = m|\theta] = p_\theta^j(m)$, $\theta \in \{-1, 1\}$, and m^j conditionally independent across sources. If coverage of source j is *not* captured by either SIG we say that the source is *honest* and in this case the information source simply publishes –i.e., truthfully conveys to their audience– the signal it observes.[18] Thus, the posterior

[18]In this model we consider honest sources which are not strategic. However, the equilibria we characterize in

belief of a $p-$citizen after observing message m from source j which is known to be honest is

$$\mu_H^j(m;p) = \Pr\left[\theta = 1 | m^j = m, H, p\right] = \frac{p_1^j(m)p}{p_1^j(m)p + p_{-1}^j(m)(1-p)}. \tag{2}$$

Without loss of generality in this binary-state case, we order messages according to the likelihood ratio $\lambda_H^j(m) = \frac{p_1^j(m)}{p_{-1}^j(m)}$ (so that $\lambda_H^j(m)$ is increasing). Following this convention, we will say that a message m is higher (lower) when citizens would update more towards state $\theta = 1$ (-1) should that message be published by a source j which is known to be honest. To characterize the informativeness of an honest source, we let $F_{H,\theta}^j(\lambda) \equiv \Pr[\lambda_H^j(m^j) \leq \lambda | \theta]$.

Competitive Capture of Information Sources: For each source j, SIGs simultaneously and covertly decide how much effort to expend in capturing the message conveyed by the source. We denote the efforts expended by R and L by $r_j \in [0, \overline{x}_R^j] \equiv X_R^j$ and $l_j \in [0, \overline{x}_L^j] \equiv X_L^j$. These efforts determine three possible states of capture, $S^j \in \{R, L, H\}$, where H indicates the source remains honest while, with a slight abuse of notation, R (L) indicates the source has been captured by the $R-$SIG $(L-$SIG). Capture is probabilistic conditional on effort, with $\pi_i^j(r_j, l_j) \equiv \Pr\left[S^j = i\right]$. We assume that $\pi_R^j(r_j, l_j)$ $(\pi_L^j(r_j, l_j))$ is continuous, non-decreasing in $r_j(l_j)$, and non-increasing in $l_j(r_j)$, while $\pi_H^j(r_j, l_j)$ is non-increasing in both arguments. Capturing efforts take resources: if $r = (r_j)_{j=1}^n$ and $l = (l_j)_{j=1}^n$ are the profiles of capturing efforts across sources, the total cost of capture for the R-SIG is $C_R(r)$ and for the L-SIG is $C_L(l)$ with C_R and C_L non-decreasing and strictly convex.

To fix ideas, in many instances we will consider a linear context function. Namely, $r_j = \Pr\left[S^j = R\right]$, $l_j = \Pr\left[S^j = L\right]$ and $1 - l_j - r_j = \Pr\left[S^j = H\right]$ with the total cost of capture for the i-SIG being $C_i(\sum_{j=1}^n \beta_j^i r_j)$, $i \in \{R, L\}$, with C_i strictly convex and satisfying standard Inada conditions.[19]

If coverage by source j is captured by either SIG, then the capturing SIG can have the source send *any* message m.[20] We assume the message space is independent of state of capture or state

Proposition 1 would also exist if honest sources were strategic and interested in the public learning the truth. See also Glazer, Herrera, and Perry (2020).

[19]In particular, we assume $C_i'(x) > 0, C_i''(x) > 0$, and $\lim_{x\to 0} C_i'(x) = 0$ and $\lim_{x \to 1} C_i'(x) = \infty$. In addition, for the linear case, we assume that $\overline{x}_R^j = \overline{x}_L^j = 1/2$ to guarantee that $0 \leq \Pr\left[S^j = i\right] \leq 1$, $i \in \{R, L, H\}$.

[20]For simplicity, we assume that the choice of message by a successful SIG is independent of media $j's$ realized

of the world so there is no restriction on the message m a captured source can convey. We allow SIGs to follow mixed strategies in deciding which messages to send. We denote by $\tau_i = (\tau_i^j(m))_{j=1}^n$, where $\tau_i^j(m) \equiv \Pr[m^j = m \,|S^j = i]$, the reporting strategy of SIG $i \in \{R, L\}$ which comprises a possibly mixed strategy over messages to be sent by each source, conditional on i capturing that source.

Information Source Audience: For clarity, we first assume that the audience of each information source –i.e., the citizens exposed to that source– is exogenous. That is, a message conveyed by source j, reaches a mass M^j of citizens whose priors are distributed according to $F_p^j(p)$, and every citizen observes the message of at most one source.[21] In Section 7, we endow citizens with a decision problem that microfounds their demand for information and we endogenize the choice of which information source to consult.

Timing: Simultaneously, SIGs R and L covertly decide on $r_j, j = 1, ..., n$ and $l_j, j = 1, ..., n$. Then, nature selects $S^j \in \{R, L, H\}$ according to $\pi_i^j(r_j, l_j)$, but neither (r_j, l_j) nor S^j are observed by citizens. For a source j such that $S^j = R$ ($S^j = L$), SIG-R (SIG-L) decides which message to send. Citizens then observe the message published by their source and update their beliefs. After this, payoffs are realized.

We look for a Perfect Bayesian Equilibrium of this capture and communication game (which we denote simply as "equilibrium"). In particular, if the $R - SIG$ selects $r = (r_j)_{j=1}^n$ and reporting strategy $\tau_R = (\tau_R^j(m))_{j=1}^n$, the $L - SIG$ selects $l = (l_j)_{j=1}^n$ and reporting strategy $\tau_L = (\tau_L^j(m))_{j=1}^n$,[22] and every citizen has an assessment of $SIGs$ strategies $(\tilde{r}, \tilde{l}, \tilde{\tau}_R, \tilde{\tau}_L)$, then every PBE $(r^*, l^*, \tau_R^*, \tau_L^*; (\tilde{r}^*, \tilde{l}^*, \tilde{\tau}_R^*, \tilde{\tau}_L^*))$ requires that citizens' assessments are correct, i.e., $\tilde{r}^* = r^*$, $\tilde{l}^* = l^*$,$\tilde{\tau}_R^* = \tau_R^*, \tilde{\tau}_L^* = \tau_L^*$, while each SIG's strategy is sequentially optimal given the other SIG's strategy and citizens' posterior beliefs, which are derived from Bayes' rule whenever possible.

This model displays a few noteworthy features. First, it focuses on the competition between SIGs and the inference problem it induces on rational consumers of information. To simplify the

signal. As we show in the online Appendix, conditioning on the realized signal does not change the distribution of viewers' posterior beliefs, nor the equilibrium capture efforts, but increases the notational burden.

[21]This single homing assumption is widespread in the literature on media bias. See, for example Gentzkow and Shapiro (2006), Chan and Suen (2008) and Duggan and Martinelli (2011) among many others.

[22]To simplify notation, we omit the reporting strategy's dependence on the selected profile of capture efforts. In any equilibrium, any reporting strategy will depend only on viewers assessments, rather than the actual level of capture.

analysis and highlight new insights, we model information sources as passive subjects of pressure from SIGs.[23] Second, we allow for multiple dimensions of heterogeneity across information sources. Specifically, sources can differ (a) in their informativeness when they remain honest $F^j_{H,\theta}(\lambda)$; (b) in the type of audience they reach $F^j_p(p)$; or (c) in how costly they are to capture by one or both SIGs β^i_j. This flexibility allows us to present general results that are compatible with traditional media, social media, scientific white papers or even religious sermons. For example, the model can accommodate the fact that some sources more frequently bias coverage towards one end of the ideological spectrum. For example, Fox News can be conceptualized as having low β^{FOX}_R so that it is cheap for the R-SIG to capture coverage at FOX. This is known by citizens, who take this into account when updating their beliefs. These citizens are asking themselves: "is FOX's coverage of this ongoing weather event honestly reporting possible links with global warming or has it (again) been compromised by the R-SIG?"

Third, messages m have an *accepted meaning* in our model, following the terminology of Sobel (2020).[24] In particular, everyone agrees how message m is to be interpreted –that is, how priors are to be updated– if a message m is published in a source which is known to be honest. The shadow of capture, however, drives a wedge between m's accepted meaning and m's interpretation *in equilibrium*. This allows us to separately keep track of published messages –i.e. equilibrium m – and the effect of such messages –i.e., equilibrium audience posteriors. This is important because, empirically, slant is reflected in m, not necessarily on viewers' posteriors.

Fourth, in interpreting the model it is important to keep in mind that the SIGs strategic choice of m may take two forms. It can bias the coverage of a given issue to suit its interests by omitting or adding details or manipulating the emphasis or emotional content. Alternatively, it can change which issues it chooses to cover, focusing on themes that are favorable to its interests. Both forms of bias have been empirically documented.[25] What is important is that in either strategy SIGs are departing from the m that would have been conveyed by the honest source, which is to be

[23] To the extent that information sources are media conglomerates, this sidesteps the media owner trade-off between audience and bias which is already well-understood in the literature.

[24] Sobel (2020) defines lies as statements whose accepted meaning is different from what the sender knows. Sources do lie along the equilibrium path in our model.

[25] See Durante, Fabiani, Laeven, and Peydro (2021) for a recent example of the former and Brookman and Kalla (2022) for a recent example of the latter.

interpreted as a composite of which issue to cover and how to cover it in order to best help the public update their beliefs.

Finally, we impose no restrictions on the message space of captured sources. More specifically, messages are not certifiable and there is no ex ante commitment to any communication strategy. In this sense we have a genuine model of lying in which capturing SIGs can have sources manufacture fake news at will, completely untethered to the true state of the world.

3 Communication Equilibria of a Captured Information Source

We start our analysis by characterizing communication equilibria for a given information source conditional on capturing efforts l and r. We present three main insights: first, potential capture leads to a polarization of expected observed messages. Second, rational citizens discount the informativeness of messages accordingly. Third, as a consequence, potential capture is very deleterious to social learning.

Since we focus on a single source, we drop for now the subscript j and we let $F_p(p)$ denote the mass of citizens with a prior at most p reached by the source.

3.1 Optimal Lying, Optimal Skepticism

Consider a citizen who observes message m published by the information source. If the source was known to be honest, the likelihood ratio $\lambda_H(m) = p_1(m)/p_{-1}(m)$ would capture the informational content of message m and would suffice to compute the posterior of a citizen with any prior p according to (2). If there is no capture, therefore, citizens interpret m according to its accepted meaning, although citizens with different priors will typically reach different posteriors. In other words, citizens agree on what message m from an honest source means –agree on $\lambda_H(m)$– but can differ on the conclusions they draw about the underlying state of the world.

The information source, however, is only honest with probability $\pi_H(r, l)$. When it is captured, m is generated according to the strategies of the capturing SIG. Consequently, m cannot be taken at face value and citizens must modify the way they update. In any communication equilibria, the lying strategies of SIGs and the updating process of citizens are consistent with each other. To describe communication equilibria, let $\tau_R^*(m)$ and $\tau_L^*(m)$ be the $R-$SIG and $L-$SIG equilibrium (mixed) strategies. These specify the probability of selecting message m if they successfully capture the information source. Let $\mu^*(m;p)$ be the posterior belief of a citizen with prior p after observing m consistent with strategies $\tau_R^*(m)$ and $\tau_L^*(m)$. Then, for $i \in \{L,R\}$ the $i-$SIG's selected message maximizes $V_i(m) = \int v_i(\mu^*(m;p))dF_p(p)$.

The following proposition shows that equilibrium behavior takes a simple form: mixing by the $R-$SIG ($L-$SIG) equalizes the *equilibrium informational content* of the highest (lowest) messages.

Proposition 1. *Consider a single information source and fix levels of capture r and l, with $\pi_H(r,l) > 0$. There are unique $\overline{\lambda}$, $\underline{\lambda}$, $\overline{m}^*$, and $\underline{m}^*$, with $\overline{\lambda} = \lambda_H(\overline{m}^*)$ and $\underline{\lambda} = \lambda_H(\underline{m}^*)$, so that for every communication equilibrium, with $\tau_R^*(m)$ and $\tau_L^*(m)$ the SIGs' equilibrium (mixed) strategies, we have*

1. *$m \in supp(\tau_R^*)$ iff $\lambda_H(m) \geq \overline{\lambda}$; $m \in supp(\tau_L^*)$ iff $\lambda_H(m) \leq \underline{\lambda}$.*
2. *The equilibrium likelihood ratio of message m, $\lambda^*(m) \equiv \frac{\Pr[m|\theta=1]}{\Pr[m|\theta=-1]}$, satisfies*

$$\lambda^*(m) = \begin{cases} \underline{\lambda} & \text{if } m \leq \underline{m}^* \\ \lambda_H(m) & \text{if } \underline{m}^* < m < \overline{m}^*. \\ \overline{\lambda} & \text{if } m \geq \overline{m}^* \end{cases} \tag{3}$$

3. *The maximum and minimum likelihood ratios $\overline{\lambda} = \max_{m \in \mathscr{M}} \lambda^*(m)$ and $\underline{\lambda} = \min_{m \in \mathscr{M}} \lambda^*(m)$ satisfy*

$$\int_{\overline{\lambda}}^{\infty} \left(\lambda - \overline{\lambda}\right) dF_{H,-1}(\lambda) = \frac{\pi_R(r,l)}{\pi_H(r,l)} \left(\overline{\lambda} - 1\right), \tag{4}$$

$$\int_0^{\underline{\lambda}} \left(\underline{\lambda} - \lambda\right) dF_{H,-1}(\lambda) = \frac{\pi_L(r,l)}{\pi_H(r,l)} \left(1 - \underline{\lambda}\right). \tag{5}$$

Part 1 of the proposition states that the $R-$SIG randomizes over a set of messages with $\lambda_H(m)$ above a threshold likelihood $\overline{\lambda}$. These are messages that would be very informative that $\theta = 1$ if sent by a source known to be honest. Part 2 describes how citizens update. For all messages possibly sent by $R-$SIG, instead of updating according to $\lambda_H(m)$, citizens just use $\overline{\lambda}$.[26] This has

[26] If we had assumed a common prior among receivers, we could express this condition in terms of the common

two implications. First, since $\overline{\lambda} \leq \lambda_H(m)$ for $m \in supp(\tau_R^*)$, the informational content of these messages is downgraded: because the source is possibly captured by $R-$SIG, citizens are skeptical of messages that are favorable to $\theta = 1$. Second, all such messages are treated identically since $\lambda^*(m) = \overline{\lambda}$, a constant. This means that the more favorable to $\theta = 1$ messages are, the stronger the downgrade that skeptical citizens apply.

Of course, the same is true at the other end of the distribution. The $L-$SIG randomizes over a set of messages favorable to state $\theta = -1$ and citizens, skeptical of such messages, treat them all as $\underline{\lambda} \geq \lambda_H(m)$. Again, they downgrade the informational content of messages below $\underline{\lambda}$ and do so more the more such messages are favorable to $\theta = -1$.

The effect of potential capture is therefore to make citizens skeptical of messages that would otherwise be very informative –i.e. either very high or very low $\lambda_H(m)$. Skepticism is well-founded because very informative messages are potential lies in equilibrium –i.e., $\Pr[S = H|m] < 1$ for such m. Moderate messages $m \in (\underline{m}^*, \overline{m}^*)$ are instead taken at face value. Upon observing them, a citizen can infer that the source is honest and updates according to $\mu^*(m;p)= \mu_H(m;p)$. The proposition thus implies that $\mu^*(m;p)$ is a two-sided censored distribution of posterior beliefs for every $p-$citizen.

Part 3 of Proposition 1 characterizes the unique $\overline{\lambda}$ and $\underline{\lambda}$ induced by a given (r,l) configuration. To build intuition note that, given a fixed level of capture, Bayesian updating requires that the equilibrium posterior beliefs of a $p-$viewer must average to the prior. Hence

$$\pi_H(r,l)\mathbb{E}_H\left[\mu^*(m;p);p\right] + \pi_L(r,l)\mu^*(\underline{m}^*;p) + \pi_R(r,l)\mu^*(\overline{m}^*;p) = p.$$

Given the two-sided censored nature of $\mu^*(m;p)$ and mixing behavior by, say, the $R-$SIG we have

$$\pi_H(r,l)\int_{\overline{m}^*}^1 \left(\mu_H(m;p) - \mu_H(\overline{m}^*;p)\right) dF_H(m;p) = \pi_R(r,l)\left(\mu_H(\overline{m}^*;p) - p\right). \qquad (6)$$

where $F_H(m;p) = pF_{H,1}(m) + (1-p)F_{H,-1}(m)$ is the distribution of messages that a $p-$receiver expects from an honest source. The left hand side of (6) represents the expected downward

posterior instead of the likelihood ratio of the message. Then, Glazer, Herrera, and Perry (2020) show that all messages sent by the biased sender generate the same posterior.

distortion in beliefs following messages from an honest source when citizens fear that the message may be captured –i.e., any $m \geq \overline{m}^*$ is suspected to come from an $R-$SIG. Conversely, the right hand side of (6) is the upward distortion by the $R-$SIG who systematically sends "high" messages. In equilibrium, the two distortions cancel each other, which determines $\overline{m}^*$. Expressing (6) in terms of likelihood ratios we obtain (4). A similar reasoning applied to the $L-$SIG leads to (5). Finally, (4) is independent of $\underline{\lambda}$ and its right hand side decreases in $\overline{\lambda}$ while its left hand side strictly increases in $\overline{\lambda}$. This implies that the solution to (4) is unique. The same argument applied to (5) yields a unique $\underline{\lambda}$. In sum, the fact that no rational viewer can be fooled in expectation, uniquely determines $\overline{\lambda}$ and $\underline{\lambda}$.

3.2 Published Messages by Captured Sources

A feature of this model is that we have predictions of the effect of capture on the (continuous) distribution of messages conveyed by a source. When the information source is known to be honest, a citizen with prior p expects each message m according to distribution $F_H(m; p)$. When the source is captured, however, the expected frequency of messages is influenced by $\tau_R^*(m)$ and $\tau_L^*(m)$. To understand how the SIGs send messages in equilibrium, note that the $R-$SIG cannot afford to exclusively send the most favorable message, which would be the highest m available. If it did, citizens would fully discount that specific message as being the result of manipulation and would update very little. Given this, the $R-$SIG could profitably deviate to sending a slightly lower message $m' = m - \epsilon$, which would induce full updating as citizens would trust that such a message could only be sent by an honest source. In equilibrium it must therefore be that the SIG randomizes over a set of messages and citizens treat all these messages equally.

To achieve this equal treatment the equilibrium mixing of each SIG distributes the probability of lying for each m in such a way that their equilibrium likelihood ratio is equalized. Note that the equilibrium likelihood ratio for a message $m \in supp(\tau_R^*)$ sent by the $R-$SIG is

$$\lambda^*(m) = \frac{\pi_H(r,l)p_1(m) + \pi_R(r,l)\tau_R^*(m)}{\pi_H(r,l)p_{-1}(m) + \pi_R(r,l)\tau_R^*(m)}$$

and this expression is decreasing in $\tau_R^*(m)$: the more often a message m is expected to be sent by the R−SIG, the less informational content citizens assign to that message.[27] Equalizing $\lambda(m)$ across the various $m \in supp(\tau_R^*)$ thus implies spreading $\tau_R^*(m)$ across messages in a very specific way that fully characterizes the optimal lying strategy of each SIG.[28]

Lemma 1. *In every communication equilibria described in Proposition 1 we have that for every two messages $m \in supp(\tau_R^*)$ and $m' \in supp(\tau_R^*)$,*

$$\begin{aligned}\tau_R^*(m')/\tau_R^*(m) &= \left(\lambda_H(m') - \overline{\lambda}\right) p_{-1}(m')/\left(\lambda_H(m) - \overline{\lambda}\right) p_{-1}(m) \\ &= \left(p_1(m') - \overline{\lambda} p_{-1}(m')\right) / \left(p_1(m) - \overline{\lambda} p_{-1}(m)\right).\end{aligned}$$

Equivalently, we have that for every two messages $m \in supp(\tau_L^*)$ and $m' \in supp(\tau_L^*)$,

$$\tau_L^*(m')/\tau_L^*(m) = (\underline{\lambda} - \lambda_H(m'))\, p_{-1}(m')/\, (\underline{\lambda} - \lambda_H(m))\, p_{-1}(m).$$

For instance, if $p_{-1}(m)$ weakly decreases in m while $p_1(m)$ increases in m, then mixing by R−SIG (L−SIG) must put more weight on higher (lower) messages in order to equalize their informational content. In other words, under such conditions, in equilibrium both SIGs send the most extreme messages relatively more often than any other message. Because these messages are more *ex ante* informative, the SIG must use them more often in order to equalize *ex post* informativeness.

Figure 1 depicts what happens to the expected distribution of published messages. Compared to the distribution that a citizen with prior p expects from an honest media (drawn in the first panel) messages from captured sources are more polarized as the mass moves towards the extreme messages that the SIGs send more frequently.

[27] This follows from the fact that the R−SIG only sends messages that make $\theta = 1$ more likely, i.e., messages such that $p_1(m) > p_{-1}(m)$.

[28] To be precise, Lemma 1 gives SIGs' optimal lying when the probability of each published message is independent of the source's honest report. See Online Appendix for the case when equilibrium lies are correlated with the source's honest report.

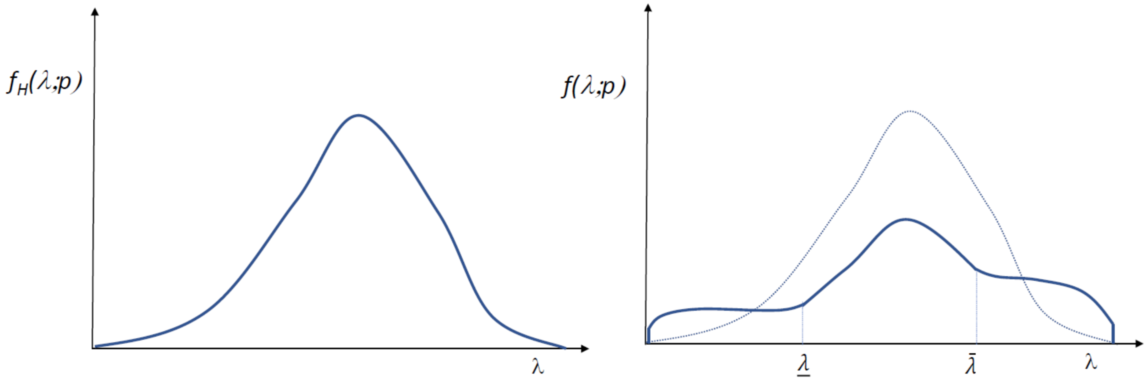

Figure 1: Content of Captured Media

3.3 Informativeness of a Captured Source

The previous discussion shows that capture affects informativeness by changing the distribution of likelihood ratios of the messages conveyed by the source. Using (3) in Proposition 1, we have that the distribution of likelihood ratios for a p-citizen is

$$F(\lambda;p) = \begin{cases} 0 & \text{if } \lambda < \underline{\lambda}, \\ \pi_L(r,l) + \pi_H(r,l)F_H(\lambda;p) & \text{if } \underline{\lambda} \leq \lambda < \overline{\lambda}, \\ 1 & \text{if } \lambda \geq \overline{\lambda}. \end{cases} \tag{7}$$

The specter of capture decreases the likelihood that a citizen revises her beliefs to entertain a very high or very low view of the world even when the message is truthful: optimal lying limits the informational content of each message to $\lambda^*(m) \in [\underline{\lambda}, \overline{\lambda}]$. As a consequence, capture reduces the Blackwell-informativeness of the source and $F(\lambda;p)$ second-order stochastically dominates $F_H(\lambda;p)$. This reduction in informativeness operates through two channels. First, it limits the informativeness of very informative messages to either $\lambda_H(\underline{m}^*) = \overline{\lambda}$ or $\lambda_H(\overline{m}^*) = \underline{\lambda}$. Second, it reduces the likelihood that a message $m \in (\underline{m}^*, \overline{m}^*)$ is observed. These two effects are depicted in Figure 2. The left hand panel shows the expected distribution of likelihood ratios associated with messages from an honest source, while the right hand side shows the expected distribution when there is possible capture by both SIGs. Contrast the right-hand panels of Figure 1 and Figure 2: while messages become polarized because of SIG interference, beliefs become compressed due to

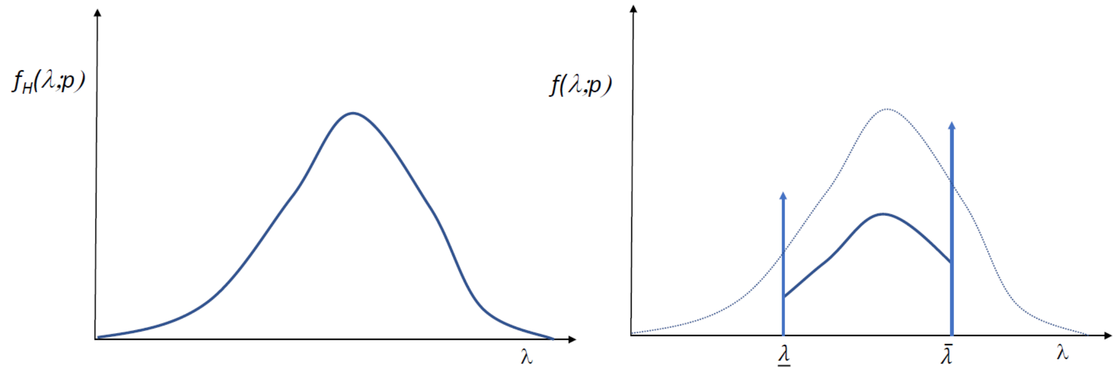

Figure 2: Informational Content with Honest and Captured Media

the skepticism generated by this interference. Just as reported in Angelucci and Prat (2021), the public discounts fake news.

We now describe how these bounds on informativeness change with exogenous changes in the parameters of the model. We show that (i) increasing capture by either SIG exacerbates citizens' skepticism over the informativeness of messages at *both* ends of the spectrum; (ii) citizens' prior distribution F_p does not affect the equilibrium informational content of the information source; and (iii) citizens discount messages less when the honest source is Blackwell more informative –i.e., when a source is more informative, equilibrium lies more successfully influence citizens.

Lemma 2. *Let $\overline{\lambda}$, $\overline{m}^*$, $\underline{\lambda}$ and $\underline{m}^*$ be the equilibrium quantities defined in Proposition 1. Then,*

1. *$\overline{\lambda}$ and $\overline{m}^*$ are decreasing in r and, if π_R/π_H increases in l, also decreasing l; while $\underline{\lambda}$ and $\underline{m}^*$ are increasing in l, and if π_L/π_H increases in r, also increasing in r.*

2. *$\overline{\lambda}$, $\overline{m}^*$, $\underline{\lambda}$ and $\underline{m}^*$ are invariant in F_p.*

3. *$\overline{\lambda}$ increases and $\underline{\lambda}$ decreases if the information source is Blackwell more informative.*

Lemma 2.1 shows that a citizen is more skeptical following any increase in capture, as it lowers the maximum (and raises the minimum) belief that she might entertain and reduces the number of messages that she will trust. Importantly, more intense capture by, say, an $R-$SIG leads citizens to trust less "favorable" reports that the state is high, but also to trust less reports that the state is low if π_L/π_H increases in r. The first effect is clear as more intense capture makes it more likely that high messages are generated by an $R-$SIG. However, an increase in capture of an $R-$SIG also makes it less likely that the source is honest. If the likelihood π_L/π_H increases, upon viewing a low m, rational citizens must place higher probability that the $L-$SIG prevailed.[29]

Lemma 2.2 shows that a source's equilibrium informativeness is invariant to its audience given l and r. This is because, as it is clear Lemma 1, equilibrium mixing equalizes the informational content of each potential message sent. As a consequence, the informational content only depends on properties of the honest source and not on the priors of the public. In short, the optimal lies of a SIG are independent of who is receiving the message. The audience of a source, however, will affect incentives to capture, as we show below.

Finally, lemma 2.3 shows that SIGs can afford to send more extreme messages if the honest source is more informative.[30] This result follows readily from a higher dispersion of posterior beliefs induced by a Blackwell more-informative honest source and its effect on equilibrium conditions (4) and (5). Intuitively, when an honest source is more informative, a given amount of lying has a smaller effect on citizens' discounting.

4 Competitive Capture of a Monopoly Information Source

Having established the effects of capture on published messages, we now turn to the determinants of equilibrium capture l and r of a monopoly information source by competing SIGs. We have two main insights. First, each SIG's marginal gain from capture is reduced when citizens expect a higher level of capture by the opposing SIG: capture efforts are strategic substitutes. Second, the effect of the distribution of citizens' priors on the incentives to capture depends on whether capturing is about firing up the base or demobilizing the opposition.

4.1 Incentives to Capture Sources

To understand the incentives to capture, the equilibrium likelihood ratio $\lambda^*(m)$ for each m suffices to characterize the distribution of viewers' posterior beliefs–see Proposition 1. By expressing each

[29] Note that these conditions $-\pi_R/\pi_H$ increasing in l and π_L/π_H increasing in $r-$ are satisfied in the linear-contest model.

[30] Note that we cannot say how this will change the messages that citizens trust as we impose no structure on the message space of a Blackwell more-informative source.

viewer's equilibrium posterior as

$$\mu^*(\lambda; p) = \frac{\lambda p}{\lambda p + 1 - p},$$

we can write the expected value to the $i-$SIG from sending a message m such that $\lambda^*(m) = \lambda$ as

$$V_i(\lambda) \equiv \int_0^1 v_i\left(\mu^*(\lambda; p)\right) dF_p(p) = \int_0^1 v_i\left(\frac{\lambda p}{\lambda p + 1 - p}\right) dF_p(p). \tag{8}$$

Note that this expression varies with the message sent –through its associated λ– and it also depends on the priors of the audience –through $F_p(p)$. To understand the equilibrium benefits of capture, consider the linear-contest model and suppose that citizens suspect a level of capture $(\tilde{r}, \tilde{l})$ and believe that the informational content of message m is $\lambda^*(m)$.[31] Increasing capture by, say, the $R-$SIG reduces the likelihood that the message originates from an honest source and substitutes the expected honest-source message with a message that is interpreted as $\overline{\lambda}$ in equilibrium. In the linear-contest model the marginal gain from capture is independent of the level of capture, and only depends on viewers' anticipated level of capture through its effect on $\overline{\lambda}$ and $\underline{\lambda}$. We can thus write the marginal gain to the $R-$SIG from covert capture as

$$\begin{aligned} B^R(\tilde{r}, \tilde{l}) = V_R\left(\overline{\lambda}\right) - \mathbb{E}_H\left[V_R(\lambda); p_R\right] = \int_{\underline{\lambda}}^{\overline{\lambda}} \left(V_R(\overline{\lambda}) - V_R(\lambda)\right) dF_H(\lambda; p_R) \\ + \left(V_R(\overline{\lambda}) - V_R(\underline{\lambda})\right) F_H(\underline{\lambda}; p_R) = \int_{\underline{\lambda}}^{\overline{\lambda}} V_R'(\lambda) F_H(\lambda; p_R) d\lambda. \end{aligned} \tag{9}$$

Note that the priors of the audience matter through $V_R'(\lambda)$, and the $R-$SIG evaluates the distribution of messages from an honest source according to her own prior belief p_R. Therefore, $B^R(\tilde{r}, \tilde{l})$ depends both on the prior beliefs of the audience and on the prior belief of the $R-$SIG. We can equivalently compute the marginal gain to the $L-$SIG as

$$B^L(\tilde{r}, \tilde{l}) = V_L\left(\underline{\lambda}\right) - \mathbb{E}_H\left[V_L(\lambda); p_L\right] = \int_{\underline{\lambda}}^{\overline{\lambda}} \left(-V_L'(\lambda)\right) \overline{F}_H(\lambda; p_L) d\lambda, \tag{10}$$

where $\overline{F}_H(\lambda; p) = 1 - F_H(\lambda; p)$.

[31] Thus, citizens' assessments $(\tilde{r}, \tilde{\tilde{l}}, \tilde{\tau}_R, \tilde{\tau}_L)$ satisfy Proposition 1 with $r = \tilde{r}$, $l = \tilde{l}$,$\tau_R^* = \tilde{\tau}_R$ and $\tau_L^* = \tilde{\tau}_L$ so that $\lambda^*(m)$ is given by (3).

4.1.1 Capturing Efforts are Strategic Substitutes

How do incentives to capture change if viewers anticipate higher capture by the other SIG? In the linear-contest model, differentiating (9) and using Lemma 2.1 we have

$$\frac{\partial B^R(\tilde{r},\tilde{l})}{\partial \tilde{l}} = V_R'(\overline{\lambda})F_H(\overline{\lambda};p_R)\frac{\partial \overline{\lambda}}{\partial \tilde{l}} - V_R'(\underline{\lambda})F_H(\underline{\lambda};p_R)\frac{\partial \underline{\lambda}}{\partial \tilde{l}} \leq 0.$$

This is one of our key insights: influence efforts are strategic substitutes. The intuition is powerful: increasing $\tilde{l}$ has a double dampening effect on the incentives of the $R-$SIG. On the one hand, the audience of the source, anticipating higher effort by the $L-$SIG become more skeptical of low messages and discount them more. Increased skepticism of low messages reduces the gains from capture by the $R-$SIG. This is captured by $\partial\underline{\lambda}/\partial\tilde{l} > 0$. On the other hand, higher $\tilde{l}$ also engenders skepticism about high messages, since they are less likely to come from an honest source. This effect is captured by $\partial\overline{\lambda}/\partial\tilde{l} < 0$. This result is an intuitive and direct corollary of Lemma 2.1. The same argument, of course, applies to the $L-$SIG.

So far we have assumed a linear contest function in which an increase in covert capture by one SIG does not crowd out influence by the other, i.e., increasing the probability that the $R-$SIG generates the message does not reduce that of an $L-$SIG. This assumption certainly strengthens the second effect which leads to $\frac{\partial\overline{\lambda}}{\partial\tilde{l}} < 0$. However, this effect is not exclusive of this formulation: strategic substitutability of influence efforts holds under more general conditions as noted in the following proposition.

Proposition 2. *Let $B^R(r;\tilde{r},\tilde{l})$ $(B^L(l;\tilde{r},\tilde{l}))$ be the marginal gain from capture to the $R - SIG$ $(L - SIG)$. Suppose that increasing i's effort weakly decreases both the probability of capture by j and the probability that the source remains honest while the ratio π_j/π_H increases. If*

$$\frac{\partial^2 \pi_i}{\partial r \partial l} = 0, \tag{11}$$

then $B^R(r;\tilde{r},\tilde{l})$ decreases in $\tilde{l}$ and $B^L(l;\tilde{r},\tilde{l})$ decreases in $\tilde{r}$.

Condition (11) allows for capture efforts by a SIG to crowd-out the opposite SIG's influence, but rules out any interaction effect with the level of effort of that SIG to avoid second order effects in order to obtain a clean result.[32] However, it is important to note that this is a sufficient condition, not a necessary one. Strategic substitutability also holds in cases where the cross-partial is non-zero, but one needs to keep track of second order effects caused by the contest function.[33]

4.2 Equilibrium Capture

The next proposition characterizes the equilibrium level of capture;

Proposition 3. *Suppose that the $i-$SIG, $i \in \{R, L\}$, can invest in capturing a source at an increasing and convex cost C_i, with capture probabilities $\pi_k(r, l)$, $k \in \{R, L, H\}$, that are concave in r and concave in l. Then, there is a pure-strategy equilibrium level of capture and, with $V_i(\lambda)$ defined in (8), every equilibrium r^* and l^* have unique $\overline{\lambda}$ and $\underline{\lambda}$ satisfying*

$$\int_{\underline{\lambda}}^{\overline{\lambda}} V_R'(\lambda) F_H(\lambda; p_R) d\lambda = C_R'(r^*), \tag{12}$$

$$\int_{\underline{\lambda}}^{\overline{\lambda}} \left(-V_L'(\lambda)\right) \overline{F}_H(\lambda; p_L) d\lambda = C_L'(l^*), \tag{13}$$

$$\int_{\overline{\lambda}}^{\infty} \left(\lambda - \overline{\lambda}\right) dF_{H,-1}(\lambda) = \frac{\pi_R(r^*, l^*)}{\pi_H(r^*, l^*)} \left(\overline{\lambda} - 1\right), \tag{14}$$

$$\int_{0}^{\underline{\lambda}} \left(\underline{\lambda} - \lambda\right) dF_{H,-1}(\lambda) = \frac{\pi_L(r^*, l^*)}{\pi_H(r^*, l^*)} \left(1 - \underline{\lambda}\right) \tag{15}$$

Equations (12-15) encapsulate the main equilibrium tension in our model: (12) and (13) show that each SIG's marginal benefit from capturing the source increases if citizens are more trusting of the source –resulting in a higher $\overline{\lambda}$ and lower $\underline{\lambda}$. Unfortunately for the SIG, more intense capture lowers citizens' trust as indicated by (14) and (15). Equations (12) and (13) imply that each SIG has no incentive to increase effort given the anticipated levels of capture while, following Proposition 1, (14) and (15) represent the most R-favorable and L-favorable equilibrium likelihood ratios consistent with expected capture.

Direct inspection of (12) shows that the marginal benefit from capture decreases if the prior belief of the $R-$sender increases: $F_H(\lambda; p_R)$ increases in a FOSD sense with increases in p_R. Thus,

[32] One such contest function would be $\pi_R(r, l) = r - \eta l, \pi_L(r, l) = l - \eta r$, with η a fixed parameter.

[33] See Corchon (2007) and Acemoglu and Jensen (2013) for treatments of the complexity of comparative statics for arbitrary contest functions.

an $R-$sender that is more optimistic of "good news" from an honest source will profit less from capture. Strategic substitutability then implies that, if the equilibrium is unique, r^* must decrease unambiguously.

In contrast, the effect of changes in audience priors is less immediate: even though $F_p(p)$ does not affect the way SIGs communicate given the anticipated level of capture –see Lemma 2– it does affect the returns from capture through its effect on the marginal gain/loss from a higher message $V_i'(\lambda)$. We explore this comparative statics in the next section.

4.3 Firing up the Base versus Demobilizing the Opposition

How SIG incentives vary with audience priors depends on the priorities of the SIG. This is intuitive: an SIG which wants to prevent the opposition from coalescing against its preferred policies needs to reach opponents and demobilize them. In contrast, an SIG which wants to incite action likely needs to prioritize already favorable citizens and radicalize them. In this section we show that our framework allows us to model this prioritization of different audience segments through features of SIG preferences.

To fix language, we say that an SIG wants to *fire up the base* if incentives to capture increase when facing a crowd of convinced partisans –i.e., low p for $L-$SIG and high p for $R-$SIG– and an SIG wants to *demobilize the opposition* if incentives are stronger with a crowd of opposite partisanship. Formally, an $R-$SIG ($L-$SIG) wants to fire up its base if $B^R (B^L)$ increases when $F_p(p)$ increases (decreases) in the FOSD sense, with a similar definition for the case in which it wants to demobilize the opposition.

From (9) and (10), audience priors affect capture incentives only through

$$V_i'(\lambda) = \int (\partial v_i(\mu(\lambda, p))/\partial\lambda)\, dF_p(p). \tag{16}$$

For $i = R$, $\partial v_R(\mu(\lambda, p))/\partial\lambda$ represents the $R-$SIG's marginal payoff from sending a more favorable message to a viewer with prior p and (16) averages this payoff across all viewers. Therefore, the $R-$SIG wants to fire up its base if $\partial v_R(\mu(\lambda, p))/\partial\lambda$ increases in p, while it wants to demobilize the opposition if $\partial v_R(\mu(\lambda, p))/\partial\lambda$ decreases in p. Likewise, the $L-$SIG wants to fire up its base (demobilize the opposition) if $-\partial v_L(\mu(\lambda,p))/\partial\lambda$ decreases (increases) in p. It follows that in both cases, an $i-$SIG wants to fire up its base if and only if $\partial v_i^2(\mu(\lambda,p))/\partial\lambda\partial p \geq 0$. The next proposition links these conditions to the curvature of v_i.

Lemma 3. *Given a source's audience F_p and its honest-reporting distribution $F_{H,\theta}$, let $[\underline{\mu},\overline{\mu}]$ be the range of posterior beliefs induced on its audience by honest coverage. There are constants $\underline{K}_i$ and $\overline{K}_i$, $i \in \{R, L\}$, such that*[34]

i-The $i-$SIG wants to fire up its base if $\frac{v_i''(\mu)}{|v_i'(\mu)|} > \underline{K}_i$, $\mu \in [\underline{\mu},\overline{\mu}]$.

ii-The $i-$SIG wants to demobilize the opposition if $\frac{v_i''(\mu)}{|v_i'(\mu)|} < \overline{K}_i$, $\mu \in [\underline{\mu},\overline{\mu}]$.

As this lemma shows, if v_i is sufficiently convex, then the SIG is mostly concerned about firing up its base, while if v_i is sufficiently concave, it mostly wants to demobilize the opposition. This is intuitive: for an $R-SIG$ the gain from raising the beliefs of the public is higher (lower) for those holding very favorable beliefs if v_R is convex (concave). The extra conditions are needed to account for the fact that a higher λ has a smaller (larger) effect on viewers posteriors if viewers hold a higher (lower) prior belief. The proof of this Lemma provides explicit expressions for the upper and lower bounds $\underline{K}_i$ and $\overline{K}_i$ on the normalized curvature which depend on characteristics of the honest source as well as audience priors. However, we show in the next lemma that convexity in the odds of a favorable state are sufficient to guarantee that SIGs want to fire up their base.

Lemma 4. *Suppose that $v_R = g_R(\mu/(1-\mu))$ and $v_L = g_L((1-\mu)/\mu)$, with g_i, $i \in \{L, R\}$, increasing and convex. Then both SIGs want to fire up their base.*

We can now proceed to analyze the effect of a shift in audience beliefs on equilibrium capture. In Proposition 4 we show that comparative statics are unambiguous when both SIG share the same motivation.

Proposition 4. *Under the conditions of Proposition 2, suppose that both SIGs want to fire-up-the-base (demobilize-the-opposition) and consider an equilibrium level of capture (r^*, l^*). If $F_p(p)$ increases (decreases) in the FOSD sense, then there is always an equilibrium $(\overline{r},\overline{l})$ with $\overline{r} \geq r^*$ and $\overline{l} \leq l^*$.*

[34]The proof of the Lemma shows that we can set $\underline{K}_R = -\overline{K}_L = K(\overline{\mu})$ and $\overline{K}_R = -\underline{K}_L = K(\underline{\mu})$ where $K(\mu) = \mu/(1-\mu) - (1-\mu)/\mu$ is the difference between the odds of $\theta = 1$ and $\theta = -1$.

The intuition behind this result is stark. If both SIGs share the same motivation, then a shift in priors must necessarily increase incentives for one and reduce them for the opponent. For example, if both SIGs want to fire up the base, a shift upwards of the distribution of beliefs in the audience moves citizens closer to the state favored by the $R-SIG$. This makes the $R-SIG$ more eager to influence the audience, since there are now more citizens in its base. At the same time, the $L-SIG$ is less interested in capture since the distribution has shifted away. These first order effects are reinforced by the strategic substitutability we describe in Proposition 2. The same reasoning yields opposite comparative statics when both SIGs want to demobilize the opposition.

5 Naive Viewers

The results we have presented so far rely fundamentally on the rational skepticism of an information source's audience. This begs the question: are these results robust to the presence of unsophisticated citizens? In this section we consider citizens with extreme susceptibility to manipulation. More precisely, we allow for a fraction $1-\gamma<1$ of citizens to be "naive" in that they believe all coverage to be honest. The remainder fraction γ of the audience are fully sophisticated as in previous sections.[35]

Naive and rational viewers interpret the same news λ differently: naive viewers take news at face value and interpret λ literally, while rational viewers are wary of capture and interpret them as $\lambda_\gamma(\lambda)$.[36] The following proposition summarizes the main features of communication equilibria with naive viewers.

Proposition 5. *In the linear-contest model, fix levels of capture r and l, with $r+l<1$, and let $V_i(\lambda)$, defined in (8), be the expected utility of the $i-SIG$ if viewers interpret the message as λ. There exists a unique equilibrium interpretation of the news by rational viewers $\lambda_\gamma(\lambda)$, with unique $\overline{\lambda}$ and $\underline{\lambda}$, satisfying*

[35]The presence of naive receivers in sender-receiver games forces strategic senders to trade-off pandering to naive receivers while making extreme messages less effective with sophisticated ones, and can lead to more informative communication (Kartik, Ottaviani, and Squintani (2007) and Chen (2011)). Closest to our model, Chen (2011) also allows for a fraction of senders to be honest. Unlike in our setup, however, all players share a common prior.

[36]To put it in terms of previous results, Proposition 1 indicates that when all viewers are rational (i.e., $\gamma=1$), $\lambda_\gamma(\lambda)=\overline{\lambda}$ for $\lambda\geq\overline{\lambda}$ while $\lambda_\gamma(\lambda)=\underline{\lambda}$ for $\lambda\leq\underline{\lambda}$.

1. $\lambda_\gamma(\lambda)$ *is given by*

$$\lambda_\gamma(\lambda) = \begin{cases} V_L^{-1}(V_L(\underline{\lambda}) + \frac{1-\gamma}{\gamma}(V_L(\underline{\lambda}) - V_L(\lambda))) & \text{if } \lambda \leq \underline{\lambda}, \\ \lambda & \text{if } \underline{\lambda} < \lambda < \overline{\lambda}, \\ V_R^{-1}(V_R(\overline{\lambda}) + \frac{1-\gamma}{\gamma}(V_R(\overline{\lambda}) - V_R(\lambda))) & \text{if } \lambda \geq \overline{\lambda}. \end{cases} \tag{17}$$

2. The associated $\overline{\lambda}$ *and* $\underline{\lambda}$ *satisfy*

$$\int_{\overline{\lambda}}^{\infty} \left(\frac{\lambda - \lambda_\gamma(\lambda)}{\lambda_\gamma(\lambda) - 1}\right) dF_{H,-1}(\lambda) = \frac{r}{1-l-r}, \tag{18}$$

$$\int_{0}^{\underline{\lambda}} \left(\frac{\lambda_\gamma(\lambda) - \lambda}{1 - \lambda_\gamma(\lambda)}\right) dF_{H,-1}(\lambda) = \frac{l}{1-l-r} \tag{19}$$

3. $\overline{\lambda}$ *decreases in* l, r, *and* γ *while* $\underline{\lambda}$ *is increasing in* l, r, *and* γ. *Fixing* $\overline{\lambda}$ *and* $\underline{\lambda}$, *then* $\lambda_\gamma(\lambda)$ *decreases (increases) in* l,r, *and* γ *for* $\lambda \geq \overline{\lambda}$ ($\lambda \leq \underline{\lambda}$).

The presence of naive citizens among the public does not qualitatively change our insights regarding message polarization and audience skepticism: the $R-$SIG selects messages with a literal meaning above some $\overline{\lambda}$ while the $L-$SIG chooses messages below $\underline{\lambda}$; this results in an increased frequency of extreme messages which, in turn, are not trusted by sophisticated citizens. However, SIGs' strategies must now balance the effect of messages on each type of citizen: as naive citizens take messages at face value, selecting messages with more favorable literal meanings must be offset by a less favorable interpretation by sophisticated citizens. This effect is captured in (17) as $\lambda_\gamma(\lambda)$ is decreasing for both $\lambda > \overline{\lambda}$ and for $\lambda < \underline{\lambda}$– see Figure 3. It follows from (17) that more extreme messages are in this model more heavily discounted by rational citizens and lead to a non-monotonic interpretation: messages whose literal reading would be more favorable are interpreted by sophisticated citizens as having less favorable implications regarding the state of the world.[37]

[37] Chen (2011) provides conditions on the constant bias in the Crawford-Sobel leading example for the existence of communication equilibria in which messages with accepted meaning are interpreted in a non-monotonic way by sophisticated receivers. In our setup, where SIGs conflict of interest is extreme, this is a feature of every communication equilibria.

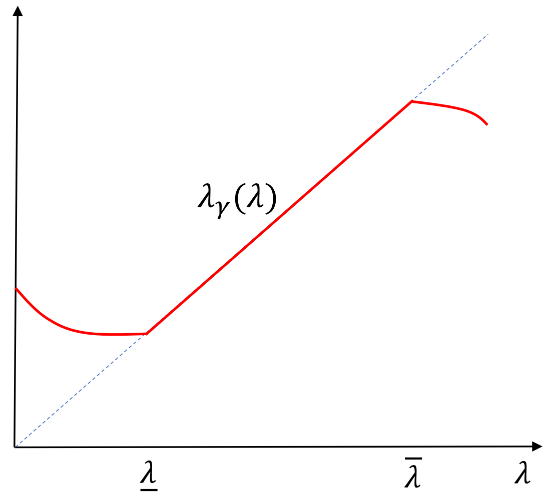

Figure 3: Equilibrium Interpretation by Sophisticated Viewers in the presence of Naive Viewers.

Another key difference between Proposition 1 and 5 is that, in the presence of naive citizens, communication equilibria can vary with the distribution of priors in the audience. The reason is that each SIG's indifference among all potential lies relies on balancing its returns from naive and sophisticated citizens, but a SIG's utility from each message interpreted at face value does depend on citizens' priors. This also implies that the highest and lowest trusted news, as given by Part 2 of the Proposition, now vary with the public's distribution of priors.

Finally, increased viewer sophistication (higher γ) makes them trust a smaller set of news – this is in part 3 of Proposition 5. This is intuitive as each SIG gains less from pandering to naive viewers. The increased need to convince sophisticated viewers means SIG must reduce the likelihood of sending the most extreme messages and therefore put more weight in more centrist messages.

A key feature of Proposition 5, as shown in part 3, is that increasing the capture level of, say, the $L-$SIG, not only reduces $\overline{\lambda}$ and increases $\underline{\lambda}$, but it also affects in a monotonic way the interpretation of the messages by sophisticated citizens: increasing l worsens the interpretation of the messages the $R-$SIG sends – by reducing $\lambda_\gamma(\lambda)$ for $\lambda \geq \overline{\lambda}$– but makes the lies of the $L-$SIG more favorable to the $R-$SIG – by increasing $\lambda_\gamma(\lambda)$ for $\lambda \leq \underline{\lambda}$. Both effects unambiguously reduce the marginal gain for the $R-$SIG from capture. Therefore, in this extended model capturing efforts are also strategic substitutes.

Proposition 6. *Suppose that there is a single information source and the probability that the*

R−SIG (L−SIG) captures the coverage is r(l). Then, for any fraction $\gamma > 0$ of sophisticated viewers, capture efforts are strategic substitutes.

This section therefore establishes that our main results, while driven by rational skepticism, are not knife-edge. Even in the presence of a large share of citizens who believe the lies they are fed, strategic and competitive SIGs must still consider how sophisticated citizens update, which leads to their efforts being strategic substitutes.

6 Competitive Capture and Polarization across Sources

We now explore several equilibrium consequences of competitive capture in the presence of multiple information sources. In this section we consider an exogenous, possibly heterogeneous, audience for each source. This allows us to analyze capture in the absence of demand-side effects coming from citizens' endogenous choice of which source to consult. In this way, our analysis sheds light on information markets where audiences' inertia or lock-in renders them unresponsive to variations in capture.[38] The next section explores endogenous source choice in response to the anticipated levels of capture.

As a preliminary result, in Appendix A we show that the existence of a pure-strategy equilibrium in capture efforts for multiple information sources is guaranteed under similar conditions as in Proposition 3. Moreover, strategic substitutability –see Proposition 2– still holds with multiple information sources when considering each individual source.

6.1 Unbalanced Capture

Differences in the intensity of capture across sources can result from vertical differences –for instance, if one source has a much larger audience than the rest, then we will expect both SIGs to intensify their efforts on that source. A similar reasoning applies to sources that are more informative when honest, or for which both SIGs have a lower cost of capture. However, casual empiricism suggests that information sources feature substantial horizontal differentiation: sources vary in their slant, with some sources heavily favoring a right-wing view of the world and others

[38]For example, Martin and McCrain (2019) suggests that audience elasticity to changes in slant brought about by changes in ownership is rather low.

favoring a left-wing view. We show in this section that strategic substitutability is a force leading in equilibrium to bigger differences in capture across sources, a form of horizontal differentiation.

To explore this mechanism, we first show that localized asymmetries in an otherwise symmetric landscape can nevertheless result in each source being differentially captured by one SIG.

Proposition 7. *Consider the linear-contest model with symmetric costs, $C_R = C_L$ and $\beta_j^R = \beta_j^L$, $j \in \{1, ..., n\}$. Suppose that there are $n-1$ symmetric information sources, with $B_j^R(\tilde{r}_j, \tilde{l}_j) = B_j^L(\tilde{r}_j, \tilde{l}_j)$ if and only if $\tilde{r}_j = \tilde{l}_j$, $j \in \{1, ..., n-1\}$. Assume source n instead is such that $B_n^R(\tilde{r}_n, \tilde{l}_n) \neq B_n^L(\tilde{r}_n, \tilde{l}_n)$ for $\tilde{r}_n = \tilde{l}_n$. Then, in every equilibrium in which source n is captured we have $r_j^* \neq l_j^*$ for every captured source $j \in \{1, ..., n-1\}$.*

In words, even if SIGs are locked into capturing $n-1$ information sources with symmetric returns, asymmetric returns in one source push SIGs to exert unbalanced efforts for every captured source. This proposition follows from the fact that each SIG equalizes marginal expected returns across all sources it tries to capture. For example, consider the $R-$SIG. If $\hat{r} = \sum_{j=1}^{n} \beta_j^R r_j^*$ is the weighted average of R's capture efforts, then we must have

$$(1/\beta_j^R) B_j^R(r_j^*, l_j^*) = (1/\beta_k^R) B_k^R(r_k^*, l_k^*) = C_R'(\hat{r})$$

whenever $r_j^*, r_k^* > 0$. Return equalization implies that changes in the returns to capturing one source affect the level of effort exerted in capturing every other source. Any horizontal difference in a source therefore has a ripple effect in equilibrium to all sources. In summary, local differences in returns to capture lead through equilibrium effects to global differences in the effort SIGs devote to each source. Therefore, we expect asymmetries in capture to be pervasive, and balanced efforts by opposed SIG for a given information source to be extremely infrequent.

6.2 Polarized Information Sources

As noted, information landscapes such as media markets tend to feature sources with polarized slants, some favoring one (partisan) view of the world while others catering to the opposite view. When is competitive capture more conducive to creating such a polarized landscape? Borrowing from spatial models of product differentiation, we first introduce two measures of polarization in capture and then analyze how these measures react to changes in the cost of capturing coverage and the distribution of audience priors.

Consider a model with two information sources and let $r = (r_1, r_2)$ and $l = (l_1, l_2)$. Our first measure of polarization, $\mathscr{P}_G(r, l)$, compares the capturing strategy by each SIG across both sources, and is defined by

$$\mathscr{P}_G(r, l) \equiv \left| \frac{r_1}{r_2} - \frac{l_1}{l_2} \right| .$$

Our second measure of polarization, $\mathscr{P}_I(r, l)$, compares the relative ideological leanings of each source stemming from capture, and is defined by

$$\mathscr{P}_I(r, l) \equiv \left| \frac{r_1}{l_1} - \frac{r_2}{l_2} \right| .$$

In both cases, we say that sources become more polarized if either $\mathscr{P}_G(r, l)$ or $\mathscr{P}_I(r, l)$ increases. While similar, these two measures have two notable differences. First, $\mathscr{P}_I(r, l)$ scales proportionally when the $R-$SIG scales all their capture efforts –that is, when the $R-$SIG switches to a strategy $\alpha r = (\alpha r_1, \alpha r_2)$ with $\alpha > 0$– but inversely in the case of scaling by the $L-$SIG. In contrast, $\mathscr{P}_G(r, l)$ controls for size effects as it is scale-invariant. Second, $\mathscr{P}_G(r, l)$ is more descriptive of differences in SIGs' behavior across sources, while $\mathscr{P}_I(r, l)$ compares the relative $R-$tilt in ideology across sources.

As the next proposition shows, local changes that affect SIGs asymmetrically can lead to more polarization under either measure.

Proposition 8. *Consider the linear-contest model with two information sources and an equilibrium level of capture (r^*, l^*) with $r_1^*/r_2^* > l_1^*/l_2^*$. Suppose that either*

a-both SIGs want to fire-up-the-base (demobilize the opposition) and $F_1(p)$ increases (decreases) in the FOSD sense, or

b-the $R-SIG'$s cost parameters change according to $\tilde{\beta}_1^R = \beta_1^R - \delta_1$ and $\tilde{\beta}_2^R = \beta_2^R + \delta_2$, $\delta_1, \delta_2 > 0$, with $\delta_2/\delta_1 = r_1^*/r_2^*$.

Then there is an equilibrium level of capture $(\overline{r}^*, \overline{l}^*)$ such that $\mathscr{P}_G(\overline{r}^*, \overline{l}^*) \geq \mathscr{P}_G(r^*, l^*)$ and $\mathscr{P}_I(\overline{r}^*, \overline{l}^*) \geq \mathscr{P}_I(r^*, l^*)$.

Local changes in source characteristics that favor the dominant SIG in that source spread in equilibrium to widen polarization across sources. To see this, consider first case (b) which describes a reduction in the relative cost of capturing source 1 for the $R-$SIG, keeping invariant the cost of capture under strategy $r^* = (r_1^*, r_2^*)$ to ensure that there are no "wealth" effects.[39] The direct effect of such cost shift leads the $R-$SIG to increase capture in source 1 and to decrease it in source 2, holding constant $L-$SIG's strategy. Strategic substitutability implies that the indirect effect generates a reinforcing response: the $L-$SIG decreases capture in source 1 and increases it in source 2. As we had $r_1^*/r_2^* > l_1^*/l_2^*$, both SIGs adjust their strategy through a rotation (increasing effort in one source, reducing it in the other) but in opposite directions, increasing both measures of polarization.

Case (a) differs from case (b) as both SIGs are directly affected by the change in audience. Consider the case in which both SIG want to fire up their base. As the audience of source 1 shifts in favor of the $R-$SIG, its incentives to capture source 1 increase at the same time that the $L-$SIG's weaken. The direct effect of the shift thus leads the $R-$SIG to increase capture in source 1, while the $L-$SIG reduces it. The effect on source 2 operates in the opposite direction as both SIG equalize expected returns. Strategic substitutability again reinforces both moves as a second order effect. Thus, we have again a rotation in the strategies of SIGs that increases media polarization.

Both cases illustrate our main insight in this Section: strategic substitutability is a force towards increased polarization across sources by amplifying local differences in the returns to capture.

7 Citizens Choice of Information Sources

In the previous Section we showed that horizontally-differentiated information sources should be expected if they are susceptible of capture by opposite SIGs. We derived this result under the proviso that audiences were exogenously fixed. In this Section, we revise our findings when allowing for the endogenous sorting of citizens across sources in response to the anticipated level of capture.

To model citizens' value of information, we endow them with the following choice problem:

[39] More specifically, it rules out the possibility that marginal costs are simultaneously reduced (or increased) for both sources after the change in cost parameters.

with probability ρ, a citizen needs to make a choice between acting ($a = 1$) and not acting ($a = 0$). For example, acting may be choosing which party to vote, going to a demonstration, or taking some decision influenced by beliefs over the seriousness of climate change. A share $1-\rho$ of citizens therefore do not have an instrumental value for information and we continue to assume that they are exogenously assigned to information sources. An increase in ρ therefore parametrizes an increase in the demand for information.

Each citizen is characterized by her prior $p = \Pr[\theta = 1]$, although it is possible to extend the analysis to a setting in which citizens also differ in ideology.[40] We use the following preferences to model this behavior: a citizen obtains 1 if $a = 1$ and $\theta = 1$, or if $a = 0$ and $\theta = -1$; and 0 otherwise. For each p we can associate $\lambda_{crit}(p)$ as follows

$$\lambda_{crit}(p) = (1-p)/p.$$

Thus $\lambda_{crit}(p)$ is the minimum likelihood ratio of a message that will lead a citizen of prior p to choose $a = 1$. For example, citizens with $p < 1/2$ –equivalently $\lambda_{crit} > 1$– do not act in the absence of news, and to act they need to see a message with informational content exceeding λ_{crit}. In contrast, citizens with $p > 1/2$ –so that $\lambda_{crit} < 1$– are already convinced of the need to act and they will only change their decision if the message's likelihood ratio falls below λ_{crit}.

First, we can show that citizens that value information sort across sources (mostly) according to their priors.

Proposition 9. *Consider the linear-contest with two symmetric sources $F_H^1 = F_H^2 (= F_H)$. Select an equilibrium with source 1 mostly captured by R−SIG (so that $r_1 \geq l_1$) and source 2 by the L−SIG (so that $l_2 \geq r_2$) while total capture is not too dissimilar in the sense that*

$$\frac{r_1}{r_2} > \frac{1-(r_1+l_1)}{1-(r_2+l_2)} > \frac{l_1}{l_2} \tag{20}$$

If $\rho = 1$, then:

[40] More specifically, we can assign to each citizen a threshold $\alpha \in (0,1)$ that her belief needs to cross for her to act. That is, if her posterior satisfies $\mu(m,p) \geq \alpha$, then the citizen chooses $a = 1$. This general case where each citizen is characterized by (p,α) is available from the authors upon request.

i-There are $\underline{p} \leq \overline{p}$ *such that citizens with* $p < \underline{p}$ *choose source 2 and citizens with* $p > \overline{p}$ *choose source 1.*

ii-If total capture is the same across outlets, $r^1 + l^1 = r^2 + l^2$*, then there is* $\tilde{p}$ *such that citizens sort monotonically: citizens choose source 2 if* $p < \tilde{p}$ *and choose source 1 if* $p > \tilde{p}$*.*

The condition (20) ensures that the odds of $R-$capture ($L-$capture) relative to an honest coverage is higher in source 1 (source 2). Citizens with low priors are reluctant to act as they believe $\theta = 1$ to be unlikely. The proposition shows that these citizens refuse to watch source 1, which is the source mostly captured by $R-$SIG, which wants to increase beliefs that $\theta = 1$. These citizens instead endogenously choose to watch source 2, which is expected to be captured by $L-$SIG, the SIG which is pushing for lower beliefs. Why do citizens choose sources that are more often captured by SIG aligned with their priors? The intuition is that low prior citizens need a strong credible message that the state is $\theta = 1$ in order to change their decision. However, source 1 is often captured by the $R-$SIG and consequently messages that favor $\theta = 1$ are suspect and not convincing enough. These citizens are better off watching source 2: if source 2 happens to remain honest, a message with high λ is possible, and coming from this source it would be credible enough for the citizens to change their choice of action.

This highlights an interesting feature of our model: the exact same message conveys different information depending on the source that publishes it. A right-wing message is therefore credible if conveyed by a left-wing source, but not credible otherwise. Because citizens with opposite priors need credibility at different ends of the message distribution, they sort accordingly: they cannot trust the messages that would be valuable to them in the information source that is often captured by the SIG that is ideologically opposed. This sorting effect is reminiscent of Suen (2004) but we obtain it in a model without filtering in which sources can freely transmit information. In fact, while in Suen (2004) bias is valuable to consumers, in our model the value of information for *all* citizens diminishes with increased capture.

However, the fact that capture reduces the value of information does not mean that increased demand for information reduces slant. The following proposition describes a situation in which the opposite is true.

Proposition 10. *Suppose that* $v_R = g(\frac{\mu}{1-\mu})$ *and* $v_L = g(\frac{1-\mu}{\mu})$ *with* g *increasing and convex and two symmetric information sources with* $F_H^1 = F_H^2 (= F_H)$. *Suppose that for* $\rho \in [0,1)$ *there is an asymmetric equilibrium with* $\bar{\lambda}_1$ $(\underline{\lambda}_2)$ *the highest (lowest) likelihood ratio in media 1 (media 2) which is dominated by R−SIG (L−SIG). Furthermore, there are two equally sized subgroups of citizens* A *and* B*, with priors satisfying*

$$p_k \geq \frac{1}{1+\underline{\epsilon}} > \frac{1}{1+\underline{\lambda}_2} \text{if } k \in A;\ p_k \leq \frac{1}{1+\bar{\epsilon}} < \frac{1}{1+\bar{\lambda}_1} \text{if } k \in B, \tag{21}$$

and citizens equally likely to consume either source if they do not value information. Then, marginally increasing ρ *increases source polarization.*

To see the intuition for this result, note that (21) implies that for any citizen in A, $\lambda_{crit}(p) \leq \underline{\epsilon} < \underline{\lambda}_2$, while any citizen in B satisfies $\lambda_{crit}(p) \geq \bar{\epsilon} > \bar{\lambda}_1$. As a marginal increase in ρ will not affect these inequalities, all viewers who value information (a proportion ρ of the population) are sorted across sources according to Proposition 9. This follows as any citizen in A will never revise its decision to act if she consumes source 2 –since capture makes potential influential messages $\lambda < \lambda_{crit}(p)(< \underline{\lambda}_2)$ not credible– and similarly for citizens in B. The rest of the audience, a fraction $1-\rho$ which do not value information, is spread equally across both sources independent of their prior.

Now consider an increase in ρ. As more citizens now value information to guide their decision, sorting increases: the proportion of citizens in A choosing source 1 and the proportion of citizens in B choosing source 2 both go up. As ρ increases therefore the $R-$SIG can reach more of the high p citizens through source 1 and less through source 2, and the opposite is true for the $L-$SIG. As g is convex, Lemma 4 establishes that SIGs want to fire up their bases. The sorting described means that the $R-$SIG can reach more of its base in source 1 (and less in source 2) and viceversa for the $L-$SIG. Both SIG thus rotate their capturing efforts in opposite directions: the $R-$SIG increases effort in 1 and reduces it in 2 and the $L-$SIG moves in opposite direction. The fact that capturing efforts are strategic substitutes further reinforces this dynamic.

As a consequence, as more citizens demand information, the system reacts with more polarization. Slant therefore increases even though the public has higher value for unbiased information.

In fact, it is easy to construct examples where citizens are worse off as a result of endogenous sorting if overall capture increases sufficiently. There are limits to this result –for example, we do not consider entry of new information sources as a result of this demand– but it is a cautionary tale on the presumption that slant is driven by lack of interest in knowing the true state of the world.

8 Conclusion

We have developed a model of competitive capture of public opinion. In the model, two opposing interest groups covertly devote effort to capture the coverage of an issue by multiple information sources who broadcast to an audience with heterogeneous priors. Captured sources can publish any fake news, untethered to the underlying state of the world, and with no commitment to any editorial line. We characterize the optimal lying strategies of special interest groups and show that capture leads to polarization in the news: extreme messages are published more often. However, rational viewers are not deceived by such messages and become skeptical. The result is deleterious to social learning as the messages that would be most informative are jammed. We also show that capturing efforts are strategic substitutes at the source level. This strategic substitution amplifies horizontal differentiation when multiple information sources are present and hence contributes to segmenting the landscape into right-leaning and left-leaning sources of information. When we allow citizens to choose which source to consult, they sort ideologically, which can reinforce horizontal differentiation if special interest groups are driven to fire up their base.

In focusing on the decisions of special interest groups, and on the informational consequences for citizens, we take a simplified view of the information sources themselves. In particular, sources are passive receivers of pressure by special interest groups and, if they remain free of capture, they are honest conveyors of information. The rich existing literature on media capture has emphasized a trade-off between profit/viewership maximizing and yielding to outside pressure which we do not consider in this model. We leave for further research to study the conditions under which this trade-off reinforces or weakens the novel mechanisms we have uncovered in this paper. In pursuing this exercise, the choice set of media owners could be enriched with actions that could enhance the reputation of the source. Indeed, the cheap talk model we have developed in this manuscript is a rich and tractable canvas which can be specialized to study multiple questions such as the targeting of audiences in social media or the effectiveness of public health campaigns as a function of the existing media landscape.

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Appendix

Proof of Proposition 1. Suppose that the $R-$SIG and $L-$SIG's strategies are $\tau_R(m)$ and $\tau_L(m)$ so that $\tau_i(m)$ is the probability that the i-SIG sends m if he captures the coverage. Then, the perceived likelihood ratio $\lambda(m) \equiv \frac{\Pr[m|\theta=1]}{\Pr[m|\theta=0]}$ is

$$\lambda(m) = \frac{\pi_H(r,l)p_1(m) + \pi_R(r,l)\tau_R(m) + \pi_L(r,l)\tau_L(m)}{\pi_H(r,l)p_{-1}(m) + \pi_R(r,l)\tau_R(m) + \pi_L(r,l)\tau_L(m)}. \tag{22}$$

The perceived likelihood ratio is sufficient to compute a p-viewer's posterior

$$\mu(m;p) = \frac{\Pr[\theta=1,m]}{\Pr[m]} = \frac{p\lambda(m)}{1-p+p\lambda(m)},$$

so that the difference in posteriors after observing two different messages m and m' is

$$\mu(m;p) - \mu(m';p) = (\lambda(m) - \lambda(m'))\frac{p(1-p)}{(1-p+p\lambda(m))(1-p+p\lambda(m'))}.$$

Averaging over the posterior of all citizens, the i-SIG's indirect utility from message m when viewers anticipate mixing $\tilde{\tau}_R(m) = \tau_R(m)$ and $\tilde{\tau}_L(m) = \tau_L(m)$ is

$$V_i(m) \equiv \int_0^1 v_i(\mu(m;p))\,dF_p(p) = \int_0^1 v_i\left(\frac{p\lambda(m)}{1-p+p\lambda(m)}\right)dF_p(p). \tag{23}$$

SIGs' optimality requires that if $m, m' \in supp\tau_i$ then $V_i(m) = V_i(m')$, $i \in \{L, R\}$. We now show that this implies that $\lambda(m) = \lambda(m')$. Indeed, suppose without loss of generality that $\lambda(m) \geq \lambda(m')$. Then, for $i = R$ we have

$$\begin{aligned}
0 &= \int_0^1 (v_R(\mu(m;p)) - v_R(\mu(m';p)))\,dF_p(p) = \int_0^1 \left(\int_{\mu(m';p)}^{\mu(m;p)} v_R'(s)\,ds\right)dF_p(p) \\
&\geq \inf_{0\leq s\leq 1}(v_R'(s))\left(\int_0^1 (\mu(m;p)) - \mu(m';p)dF_p(p)\right) \\
&= \inf_{0\leq s\leq 1}(v_R'(s))\,(\lambda(m) - \lambda(m'))\int_0^1\left(\frac{p(1-p)}{(1-p+p\lambda(m))(1-p+p(\lambda(m'))}\right)dF_p(p) \\
&\geq 0
\end{aligned}$$

as the integrand in the last equation is strictly positive. Since v_R' is bounded away from zero, we must then have that $\lambda(m) = \lambda(m')$. A similar argument would establish that $\lambda(m) = \lambda(m')$ if $m, m' \in$ *support* τ_L.

Note that (a) $V_R(m)$ in (23) is strictly increasing in $\lambda(m)$ while $V_L(m)$ in (23) is strictly decreasing in $\lambda(m)$, and (b) if $\tau_R(m) = \tau_L(m) = 0$ then $\lambda(m) = \lambda_H(m)$. Letting $\lambda^*(m)$ be the equilibrium likelihood ratio of message m with $\overline{\lambda} = \max_{m \in \mathscr{M}} \lambda^*(m)$ and $\underline{\lambda} = \min_{m \in \mathscr{M}} \lambda^*(m)$, then $\lambda^*(m) = \overline{\lambda}$ if $m \in supp(\tau_R^*)$ while (ii) implies that $m \in supp(\tau_R^*)$ only if $\lambda_H(m) \geq \overline{\lambda}$. If $\overline{m}^*$ is defined by $\lambda_H(\overline{m}^*) = \overline{\lambda}$ then $m \in supp(\tau_R^*)$ iff $m \geq \overline{m}^*$. Conversely, if $m \in supp(\tau_L^*)$ then $\lambda^*(m) = \underline{\lambda}$ and $m \in supp(\tau_L^*)$ iff $\lambda_H(m) \leq \underline{\lambda}$. Thus, if $\underline{m}^*$ is defined by $\lambda_H(\underline{m}^*) = \underline{\lambda}$, then $m \in supp(\tau_R^*)$ iff $m \leq \underline{m}^*$.

Note that, generically, the R and L lobbyists will never send the same message with positive probability –this will always be the case if $\pi_H(r, l) > 0$. In this case, we must have in equilibrium that $\tau_R^*(m)\tau_L^*(m) = 0$ for all $m \in \mathscr{M}$.

Using (22) we can write for all m such that $\lambda_H(m) \geq \overline{\lambda}$

$$\frac{\pi_R(r, l)}{\pi_H(r, l)} \left(\overline{\lambda}\tau_R(m) - \tau_R(m)\right) = \left(\lambda_H(m) - \overline{\lambda}\right) p_{-1}(m), \tag{24}$$

and for all m such that $\lambda_H(m) \leq \underline{\lambda}$

$$\frac{\pi_L(r, l)}{\pi_H(r, l)} \left(\tau_L(m) - \underline{\lambda}\tau_L(m)\right) = \left(\underline{\lambda} - \lambda_H(m)\right) p_{-1}(m). \tag{25}$$

Integrating (24) over $\left\{m : \lambda_H(m) \geq \overline{\lambda}\right\}$ gives (4). A similar argument yields (5) from (25). The right hand-side of (4) is increasing, and the left hand side is non-increasing, in $\overline{\lambda}$, thus, guaranteeing a unique solution to (4). The same argument establishes uniqueness of $\underline{\lambda}$ satisfying (5) $\square$

Proof of Lemma 1. We can solve for $\tau_R^*(m)$ and $\tau_L^*(m)$ using (22) with $\tau_R(m) = \tau_R^*(m)$ and

$\tau_R(m) = \tau_L^*(m)$ to obtain

$$\tau_R^*(m) = \frac{\pi_H(r,l)}{\pi_R(r,l)} \left(\frac{\lambda_H(m) - \overline{\lambda}}{\overline{\lambda} - 1} \right) p_{-1}(m),$$
$$\tau_L^*(m) = \frac{\pi_H(r,l)}{\pi_L(r,l)} \left(\frac{\underline{\lambda} - \lambda_H(m)}{1 - \underline{\lambda}} \right) p_{-1}(m),$$

implying that $\tau_R^*(m')/\tau_R^*(m) = \left(\lambda_H(m') - \overline{\lambda}\right) p_{-1}(m')/ \left(\lambda_H(m) - \overline{\lambda}\right) p_{-1}(m)$ and $\tau_L^*(m')/\tau_L^*(m) = \left(\underline{\lambda} - \lambda_H(m')\right) p_{-1}(m')/ \left(\underline{\lambda} - \lambda_H(m)\right) p_{-1}(m)$. □

Proof of Lemma 2. (1) Note that $\pi_R(r,l)/\pi_H(r,l)$ increases in r and $\pi_L(r,l)/\pi_H(r,l)$ increases in l. This implies that the right hand sides of (4) and (5) increase with r and l, respectively. Equilibrium then requires that $\overline{\lambda}$ must decrease (as well as $\overline{m}^*$) with r, while $\underline{\lambda}$ must increase (as well as $\underline{m}^*$) with l. The same argument applies to changes in l in (4) and in r in (5) under the condition that π_R/π_H increases in l, and π_L/π_H increases in r.

(2) Proposition 1 shows that $\overline{\lambda}$, $\overline{m}^*$, $\underline{\lambda}$ and $\underline{m}^*$ do not vary with F_p as the equilibrium conditions (4) and (5) do not depend on citizens' prior distribution.

(3) To prove that $\overline{\lambda}$ increases and $\underline{\lambda}$ decreases when the honest sender is Blackwell-more informative, we will exploit the fact that posterior beliefs are more disperse (in the sense of second order stochastic dominance) under the more informative sender (Blackwell and Girshick (1954)). To do this, we will express (4) and (5) in terms of posterior beliefs $\mu(m;p)$ for $p \in (0,1)$. First, we can write

$$\begin{aligned}
\frac{\lambda_H(m) - \lambda_H(\overline{m}^*)}{\lambda_H(\overline{m}^*) - 1} p_{-1}(m) &= \frac{1}{\lambda_H(\overline{m}^*) - 1} p_1(m) - \frac{\lambda_H(\overline{m}^*)}{\lambda_H(\overline{m}^*) - 1} p_{-1}(m) \\
&= \frac{p(1 - \mu_H(\overline{m}^*;p))}{\mu_H(\overline{m}^*;p)) - p} p_1(m) - \frac{\mu_H(\overline{m}^*;p)(1-p)}{\mu_H(\overline{m}^*;p)) - p} p_{-1}(m) \\
&= \left(\frac{\mu_H(m;p)) - \mu_H(\overline{m}^*;p))}{\mu_H(\overline{m}^*;p)) - p} \right) \Omega_H(m;p)
\end{aligned}$$

with $\Omega_H(m;p) \equiv p_1(m)p + p_{-1}(m)(1-p)$ the $p-$citizen probability of observing m by an honest media. Then, (4) can be expressed as

$$\int_{\{m:\mu_H(m;p)\geq\overline{\mu}(p)\}} (\mu_H(m;p)) - \overline{\mu}(p))\,\Omega_H(m;p)dm = \frac{\pi_R(r,l)}{\pi_H(r,l)}\left(\overline{\mu}(p)-1\right)$$

where $\overline{\mu}(p) \equiv \mu_H(\overline{m}^*;p))$. Integrating by parts and expressing the result in terms of $\mu_H = \mu_H(m;p)$ we can write

$$\int_{\overline{\mu}(p)}^{1} \overline{F}_H(\mu_H;p)d\mu_H = \frac{\pi_R(r,l)}{\pi_H(r,l)}\left(\overline{\mu}(p)-p\right) \tag{26}$$

If honest media H' is Blackwell-more informative than honest media H, then Blackwell and Girshick (1954) shows that for every $p \in (0,1)$

$$\int_{\overline{\mu}(p)}^{1} \overline{F}_{H'}(\mu_{H'};p)d\mu_{H'} \geqq \int_{\overline{\mu}(p)}^{1} \overline{F}_H(\mu_H;p)d\mu_H,$$

so that to satisfy (26), we must have a higher maximum belief in equilibrium under H'. This implies that $\lambda_H(\overline{m}^*)$ must increase. Conversely, from

$$\frac{\lambda_H(\underline{m}^*)-\lambda_H(m)}{1-\lambda_H(\underline{m}^*)}p_{-1}(m) = \left(\frac{\mu_H(\underline{m}^*;p)-\mu_H(m;p)}{p-\mu_H(\underline{m}^*;p)}\right)\Omega_H(m;p)$$

we have that (5) translates, after integrating by parts, to

$$\int_{0}^{\underline{\mu}(p)} F_H(\mu_H;p)d\mu_H = \frac{\pi_L(r,l)}{\pi_H(r,l)}\left(p-\underline{\mu}(p)\right) \tag{27}$$

where $\underline{\mu}(p) = \mu_H(\underline{m}^*;p))$. A Blackwell-more informative sender satisfies

$$\int_{0}^{\underline{\mu}(p)} F_{H'}(\mu_{H'};p)d\mu_{H'} \geq \int_{0}^{\underline{\mu}(p)} F_H(\mu_H;p)d\mu_H$$

so that $\underline{\mu}$ must decrease to satisfy (27), implying a lower $\lambda_H(\underline{m}^*) = \underline{\lambda}$. □

Proof of Proposition 2. Suppose that citizens anticipate $(\tilde{r},\tilde{l},\tilde{\tau}_R,\tilde{\tau}_L)$ with $(\tilde{\tau}_R,\tilde{\tau}_L)$ satisfying Proposition 1 with $r=\tilde{r}$ and $l=\tilde{l}$; a $p-$citizen's posterior belief after observing m is $\mu^*(m;p) = \frac{\lambda^*(m)p}{\lambda^*(m)p+1-p}$ with $\lambda^*(m)$ satisfying Proposition 1.2, where thresholds $\overline{\lambda}$ and $\underline{\lambda}$ are determined by (4) and (5); and the $i-$SIG's interim utility from sending message m with likelihood $\lambda = \lambda^*(m)$ is

$$V_i(\lambda) \equiv \int v_i\left(\mu^*(m;p)\right) dF_p(p) = \int v_i\left(\frac{\lambda p}{\lambda p + 1 - p}\right) dF_p(p),$$

Then, the $R-$SIG and $L-$SIG's expected utility when investing r and l in covertly capturing the source, followed by a sequentially rational reporting strategy, are

$$W_R(r,l;\tilde{r},\tilde{l}) = \pi_R(r,l)V_R(\overline{\lambda}) + \pi_L(r,l)V_R(\underline{\lambda}) + \pi_H(r,l)\mathbb{E}_H\left[V_R(\lambda);p_R\right] - C_R(r), \tag{28}$$

$$W_L(r,l;\tilde{r},\tilde{l}) = \pi_R(r,l)V_L(\overline{\lambda}) + \pi_L(r,l)V_L(\underline{\lambda}) + \pi_H(r,l)\mathbb{E}_H\left[V_L(\lambda);p_R\right] - C_L(l). \tag{29}$$

Focusing on the $R-$SIG, he evaluates the likelihood that an honest source would have sent a message inducing $\lambda = \lambda^*(m)$ according to his prior p_R, so that

$$\mathbb{E}_H\left[V_R(\lambda);p_R\right] = \overline{F}_H(\overline{\lambda};p_R)V_R(\overline{\lambda}) + \int_{\underline{\lambda}}^{\overline{\lambda}} V_R(\lambda)dF_H(\lambda;p_R) + F_H(\underline{\lambda};p_R)V_R(\underline{\lambda}). \tag{30}$$

Therefore, the $R-$SIG 's marginal gain from covertly increasing source capture is

$$\frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r} = \frac{\partial \pi_R(r,l)}{\partial r}V_R(\overline{\lambda}) + \frac{\partial \pi_L(r,l)}{\partial r}V_R(\underline{\lambda}) + \frac{\partial \pi_H(r,l)}{\partial r}\mathbb{E}_H\left[V_R(\lambda);p_R\right]$$

as citizens' interpretation of messages only depends on the expected level of capture $(\tilde{r},\tilde{l})$ rather than the actual level (r,l). Let $B^R(r;\tilde{r},\tilde{l})$ be the $R-$SIG's marginal gain when citizens correctly anticipate the $L-$SIG's capture effort –i.e., when $\tilde{l} = l$. Then, the change in $B^R(r;\tilde{r},\tilde{l})$ if the $L-$SIG increases its level of capture and it is correctly anticipated by viewers is

$$\frac{\partial B^R(r;\tilde{r},\tilde{l})}{\partial \tilde{l}} = \frac{\partial^2 W_R(r,l;\tilde{r},\tilde{l})}{\partial r \partial l} + \left.\frac{\partial^2 W_R(r,l;\tilde{r},\tilde{l})}{\partial r \partial \tilde{l}}\right|_{l=\tilde{l}}$$

$$= \left.\frac{\partial^2 \pi_R(r,l)}{\partial r \partial l}\right|_{l=\tilde{l}} V_R(\overline{\lambda}) + \left.\frac{\partial \pi_L^2(r,l)}{\partial r \partial l}\right|_{l=\tilde{l}} V_R(\underline{\lambda}) + \left.\frac{\partial^2 \pi_H(r,l)}{\partial r \partial l}\right|_{l=\tilde{l}} \mathbb{E}_H\left[V_R(\lambda);p_R\right]$$

$$` + \left.\frac{\partial \pi_R(r,l)}{\partial r}\right|_{l=\tilde{l}} V_R'(\overline{\lambda})\frac{\partial \overline{\lambda}}{\partial \tilde{l}} + \left.\frac{\partial \pi_L(r,l)}{\partial r}\right|_{l=\tilde{l}} V_R'(\underline{\lambda})\frac{\partial \underline{\lambda}}{\partial \tilde{l}} + \left.\frac{\partial \pi_H(r,l)}{\partial r}\right|_{l=\tilde{l}} \frac{\partial \mathbb{E}_H\left[V_R(\lambda);p_R\right]}{\partial \tilde{l}}.$$

Differentiating (30) we have

$$\frac{\partial \mathbb{E}_H\left[V_R(\lambda);p_R\right]}{\partial \widetilde{l}} = \overline{F}_H(\overline{\lambda};p_R)V_R'(\overline{\lambda})\frac{\partial \overline{\lambda}}{\partial \widetilde{l}} + F_H(\underline{\lambda};p_R)V_R'(\underline{\lambda})\frac{\partial \underline{\lambda}}{\partial \widetilde{l}},$$

and using the assumption that $\frac{\partial^2 \pi_i(r,l)}{\partial r \partial l} = 0$ we have

$$\frac{\partial B^R(r;\tilde{r},\tilde{l})}{\partial \widetilde{l}} = \left(\left.\frac{\partial \pi_R(r,l)}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H(r,l)}{\partial r}\right|_{l=\tilde{l}} \overline{F}_H(\overline{\lambda};p_R) \right) V_R'(\underline{\lambda})\frac{\partial \overline{\lambda}}{\partial \widetilde{l}} \tag{31}$$

$$+ \left(\left.\frac{\partial \pi_L(r,l)}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H(r,l)}{\partial r}\right|_{l=\tilde{l}} F_H(\underline{\lambda};p_R) \right) V_R'(\underline{\lambda})\frac{\partial \underline{\lambda}}{\partial \widetilde{l}}. \tag{32}$$

We now show that $\partial B^R(r;\tilde{r},\tilde{l})/\partial\widetilde{l} \leq 0$ so the $R-$SIG's capture incentives decrease with the anticipated level of capture of the $L-$SIG. Since $\sum_{i\in\{H,R,L\}} \pi_i(r,l) = 1$, then $\sum_{i\in\{H,R,L\}} \partial\pi_i(r,l)/\partial r = 0$ and, by assumption, $\frac{\partial \pi_H(r,\widetilde{l})}{\partial r} \leq 0$ and $\frac{\partial \pi_L(r,\widetilde{l})}{\partial r} \leq 0$. Therefore, we must have $\partial\pi_R(r,l)/\partial r = |\partial\pi_H(r,l)/\partial r| + |\partial\pi_L(r,l)/\partial r|$ so that the first term in parenthesis in (31) is positive while the term in parenthesis in (32) is negative. From lemma 2.1, given that $\pi_R(r,\widetilde{l})/\pi_H(r,\widetilde{l})$ increases in $\widetilde{l}$, the effect of increasing $L-$capture is to decrease $\overline{\lambda}$ and increase $\underline{\lambda}$. Therefore, $\partial B^R(r;\tilde{r},\tilde{l})/\partial\widetilde{l}$ must be negative.

A similar analysis applied to capture by the $L-$SIG shows that $\partial B^L(l;\tilde{r},\tilde{l})/\partial\widetilde{r} \leq 0$. □

Proof of Proposition 3. Recall that the *SIGs* expected utility when viewers anticipate capture levels $(\widetilde{r},\widetilde{l})$ is (28) and (29) with $\overline{\lambda}$ and $\underline{\lambda}$ consistent with $(\widetilde{r},\widetilde{l})$ –i.e., satisfying (4) and (5). Proposition 11 in the Appendix establishes existence of a pure-strategy equilibrium in capture efforts when $\pi_i(r,l)$ are concave in r and concave in l. In any such equilibrium, we must have

$$r^* \in \underset{r\in X_R}{\arg\max} W_R(r,l^*;(r^*,l^*))$$

$$l^* \in \underset{l\in X_L}{\arg\max} W_L(r^*,l;(r^*,l^*)).$$

Using (9) and (10), we can express these equilibrium conditions as (12) and (13). As citizens correctly anticipate (r^*,l^*), then (4) and (5) provides the equilibrium maximum and minimum likelihood ratio. □

Proof of Lemma 3. With $\mu = \mu(\lambda, p)$ to simplify notation, we show that under (i), $\partial^2 v_i(\mu)/\partial\lambda\partial p > 0$ so that the $i-$SIG wants to fire up its base, while under (ii) we have $\partial^2 v_i(\mu)/\partial\lambda\partial p < 0$ so that the $i-$SIG wants to demobilize its opposition. First, differentiating $v_i(\mu)$

$$\frac{\partial^2 v_i(\mu)}{\partial\lambda\partial p} = v_i''(\mu)\frac{\partial\mu}{\partial\lambda}\frac{\partial\mu}{\partial p} + v_i'(\mu)\frac{\partial^2\mu}{\partial\lambda\partial p}.$$

We have $\frac{\partial\mu}{\partial\lambda} = \frac{p(1-p)}{(\lambda p+1-p)^2}$, $\frac{\partial\mu}{\partial p} = \frac{\lambda}{(\lambda p+1-p)^2}$ and $\frac{\partial^2\mu}{\partial\lambda\partial p} = \frac{1-p-\lambda p}{(\lambda p+1-p)^3}$ so that

$$\begin{aligned}
\frac{\partial^2 v_i(\mu)}{\partial\lambda\partial p} &= v_i''(\mu)\frac{\lambda p(1-p)}{(\lambda p+1-p)^4} + v_i'(\mu)\frac{1-p-\lambda p}{(\lambda p+1-p)^3} \\
&= \frac{\lambda p(1-p)}{(\lambda p+1-p)^4}\left(v_i''(\mu) + \frac{(1-p-\lambda p)(\lambda p+1-p)}{\lambda p(1-p)}v_i'(\mu)\right) \\
&= \frac{\lambda p(1-p)}{(\lambda p+1-p)^4}\left(v_i''(\mu) - K(\mu)v_i'(\mu)\right),
\end{aligned}$$

with

$$K(\mu) = \frac{\lambda p}{1-p} - \frac{1-p}{\lambda p} = \frac{\mu}{1-\mu} - \frac{1-\mu}{\mu},$$

the difference between the odds of a high state and a low state. As $K(\mu)$ is increasing in μ, we have $K(\mu) \in \left[K(\underline{\mu}), K(\overline{\mu})\right]$ with $[\underline{\mu}, \overline{\mu}]$ the range of posteriors induced on citizens when consuming the the coverage of a source known to be honest.

Consider first the case of the $R-$SIG. As $v_R'(\mu) > 0$, then $\partial^2 v_R(\mu)/\partial\lambda\partial p > 0$ if $\min_{\mu\in[\underline{\mu},\overline{\mu}]} \frac{v_R''(\mu)}{v_R'(\mu)} > \max_{\mu\in[\underline{\mu},\overline{\mu}]} K(\mu) = K(\overline{\mu})$ while $\partial^2 v_R(\mu)/\partial\lambda\partial p < 0$ if $\max_{\mu\in[\underline{\mu},\overline{\mu}]} \frac{v_R''(\mu)}{v_R'(\mu)} < \min_{\mu\in[\underline{\mu},\overline{\mu}]} K(\mu) = K(\underline{\mu})$. Turning to the $L-$SIG, we have $v_L'(\mu) < 0$ so that $\partial^2 v_L(\mu)/\partial\lambda\partial p > 0$ if $\min_{\mu\in[\underline{\mu},\overline{\mu}]} \frac{v_L''(\mu)}{|v_L'(\mu)|} > \max_{\mu\in[\underline{\mu},\overline{\mu}]} -K(\mu) = -K(\underline{\mu})$ while $\partial^2 v_L(\mu)/\partial\lambda\partial p < 0$ if $\max_{\mu\in[\underline{\mu},\overline{\mu}]} \frac{v_L''(\mu)}{|v_L'(\mu)|} < \min_{\mu\in[\underline{\mu},\overline{\mu}]} -K(\mu) = -K(\overline{\mu})$. □

Proof of Lemma 4. We can express the odds of the high state as $\mu/(1-\mu) = \lambda p/(1-p)$. Then,

$$\begin{aligned}
\frac{\partial^2 v_R(\mu)}{\partial\lambda\partial p} &= \frac{1}{(1-p)^2}\left(g_R''\left(\frac{\lambda p}{1-p}\right)\frac{\lambda p}{1-p} + g_R'\left(\frac{\lambda p}{1-p}\right)\right) \\
&= \frac{1}{(1-p)^2}\left.\frac{d\left(g_R'(x)x\right)}{dx}\right|_{x=\frac{\lambda p}{1-p}}.
\end{aligned}$$

If $g'_R(x)x$ is increasing, then the $R-$SIG wants to fire up its base, while he wants to demobilize the opposition if $g'_R(x)x$ is decreasing. A sufficient condition for an increasing $g'_R(x)x$ is that g_R is convex. The same analysis applies to the $L-$SIG once we observe that

$$\begin{aligned}\frac{\partial^2 v_L(\mu)}{\partial\lambda\partial p} &= \frac{1}{\lambda^2 p^2}\left(g''_L\left(\frac{1-p}{\lambda p}\right)\frac{1-p}{\lambda p} + g'_L\left(\frac{1-p}{\lambda p}\right)\right) \geq 0 \\ &= \frac{1}{\lambda^2 p^2}\left.\frac{d\left(g'_L(x)x\right)}{dx}\right|_{x=\frac{1-p}{\lambda p}}.\end{aligned}$$

□

Proof of Proposition 4. As defined in (28) and (29), let $W_R(r,l;\tilde{r},\tilde{l})$ and $W_L(r,l;\tilde{r},\tilde{l})$ be the $R-SIG$ and $L-SIG's$ expected utility when citizens anticipate capture levels $(\widetilde{r},\widetilde{l})$. Define the $i-SIG$'s best-response function given citizens' assessment of capture efforts,

$$\tilde{\Psi}_R(l;\tilde{r},\tilde{l}) \equiv \{r : W_R(r,l;\tilde{r},\tilde{l}) \geq W_R(r',l;\tilde{r},\tilde{l}), r' \in X_R\}, \tag{33}$$

$$\tilde{\Psi}_L(r;\tilde{r},\tilde{l}) \equiv \{l : W_L(r,l;\tilde{r},\tilde{l}) \geq W_L(r,l';\tilde{r},\tilde{l}), l' \in X_L\}. \tag{34}$$

The fact that $W_R(\cdot,l;\tilde{r},\tilde{l})$ $(W_L(r,\cdot;\tilde{r},\tilde{l}))$ is strictly concave in $r(l)$ guarantees a unique maximizer, and thus $\tilde{\Psi}_R(l;\tilde{r},\tilde{l})(\tilde{\Psi}_L(r;\tilde{r},\tilde{l}))$ is indeed a function. Define

$$\hat{\Psi}_R(l) = \{r : r = \tilde{\Psi}_R(l;r,l), r \in X_R\},$$

$$\hat{\Psi}_L(r) = \{l : r = \tilde{\Psi}_L(r;r,l), l \in X_L\}.$$

For instance, $\hat{\Psi}_R(l)$ is the belief-consistent best response by the $R-$SIG when citizens correctly anticipate the $L-$SIG playing l –i.e., $\hat{\Psi}_R(l)$ is the set of fixed points $r = \tilde{\Psi}_L(l;r,l)$ parametrized by l. We can similarly interpret $\hat{\Psi}_L(r)$. Observe that $\hat{\Psi}_R(l)$ and $\hat{\Psi}_R(l)$ are functions. The fact that they are non-empty follows from (i) applying Brouwer's fixed-point theorem to the continuous function $\tilde{\Psi}_R(l;\cdot,l)$ and $\tilde{\Psi}_L(r;r,\cdot)$ –see Proposition 11 for the proof of continuity– to prove existence, and (ii) the uniqueness of solution to $r = \tilde{\Psi}_R(l;r,l)$ $(l = \tilde{\Psi}_L(r;r,l))$ follows from $\tilde{\Psi}_R(l;\cdot,l)$ $(\tilde{\Psi}_L(r;r,\cdot))$ being non-increasing. Two final remarks: (a) $\hat{\Psi}_R(l)$ and $\hat{\Psi}_R(l)$ are non-increasing under the conditions in

Proposition 2, a consequence of strategic substitutability, and (b) (r^*, l^*) is an equilibrium profile of capture efforts if and only if $r^* = (\hat{\Psi}_R \circ \hat{\Psi}_L)(r^*)$ and $l^* = (\hat{\Psi}_L \circ \hat{\Psi}_R)(l^*)$.

Suppose that F_p increases in the FOSD sense and let $\tilde{\Psi}^{\delta}_R(l; r, l)$, $\tilde{\Psi}^{\delta}_L(r; r, l)$, $\hat{\Psi}^{\delta}_R(l)$, and $\hat{\Psi}^{\delta}_L(r)$ be the corresponding functions after the change in the audience reach. As both SIGs want to fire up their base, the change in F_p raises the marginal gain from capture to the $R-$SIG, so $\tilde{\Psi}^{\delta}_R(l; r, l) \geq \tilde{\Psi}_R(l; r, l)$, and lowers that of the $L-$SIG, so $\tilde{\Psi}^{\delta}_L(r; r, l) \leq \tilde{\Psi}_L(r; r, l)$, implying that $\hat{\Psi}^{\delta}_R(l) \geq \hat{\Psi}_R(l)$ and $\hat{\Psi}^{\delta}_L(r) \leq \hat{\Psi}_L(r)$. But then,

$$\hat{\Psi}^{\delta}_R(\hat{\Psi}^{\delta}_L(r)) \geq \hat{\Psi}_R(\hat{\Psi}^{\delta}_L(r)) \geq \hat{\Psi}_R(\hat{\Psi}_L(r)),$$

where the last inequality follows from $\hat{\Psi}_R(\cdot)$ being non-increasing. Likewise, we have

$$\hat{\Psi}^{\delta}_L(\hat{\Psi}^{\delta}_R(l)) \leq \hat{\Psi}_L(\hat{\Psi}^{\delta}_R(l)) \leq \hat{\Psi}_L(\hat{\Psi}_R(l)),$$

where the last inequality follows from $\hat{\Psi}_L(\cdot)$ being non-increasing. Taking together, this implies that the highest fixed point of $\hat{\Psi^{\delta}}_R \circ \hat{\Psi^{\delta}}_L$ is higher than the highest fixed point of $\hat{\Psi}_R \circ \hat{\Psi}_L$; while the lowest fixed point of $\hat{\Psi^{\delta}}_L \circ \hat{\Psi^{\delta}}_R$ is lower than the lowest fixed point of $\hat{\Psi}_L \circ \hat{\Psi}_R$– see Villas-Boas (1997).

Finally, let $\overline{r} = \max\{r \in X_R : r = \hat{\Psi}^{\delta}_R \circ \hat{\Psi}^{\delta}_L(r)\}$ with $\bar{l} = \hat{\Psi}^{\delta}_L(\overline{r})$. For any equilibrium (r^*, l^*) before the change in the reach of the audience, we have shown that $r^* \leq \overline{r}$. We now show that $\bar{l} \leq l^*$. Indeed,

$$\bar{l} = \hat{\Psi}^{\delta}_L(\overline{r}) \leq \hat{\Psi}_L(\overline{r}) \leq \hat{\Psi}_L(r^*) = l^*$$

where the first inequality follows from the decrease in the marginal gain to the $L-$SIG and the second inequality from $\hat{\Psi}_L$ being non-increasing.

The case of a FOSD decrease in F_p when SIGs want to demobilize the opposition follows along similar lines. □

Proof of Proposition 5. Suppose that the sophisticated citizens' assessments of the reporting strategies of $R-$SIG and $L-$SIG's strategies, expressed in terms of the accepted meaning, are

$\tau_R(\lambda)$ and $\tau_L(\lambda)$. Then, the perceived likelihood ratio by sophisticated viewers, $\lambda_\gamma(\lambda) \equiv \frac{\Pr[\lambda|\theta=1]}{\Pr[\lambda|\theta=0]}$, is

$$\lambda_\gamma(\lambda) = \frac{(1-l-r)p_1(\lambda) + r\tau_R(\lambda) + l\tau_L(\lambda)}{(1-l-r)p_{-1}(\lambda) + r\tau_R(\lambda) + l\tau_L(\lambda)}, \tag{35}$$

while the $i-$SIG's expected utility from a message that is interpreted as λ is $V_i(\lambda)$ as given by (8). Then, the expected utility of the $i-$SIG when sending a message with literal meaning λ is

$$\tilde{V}_i(\lambda) \equiv (1-\gamma)\, V_i(\lambda) + \gamma V_i(\lambda_\gamma(\lambda)). \tag{36}$$

If SIGs select $\tau_R(\lambda)$ and $\tau_L(\lambda)$, $i-$SIG's optimality, $i \in \{L, R\}$, requires that if $\lambda, \lambda' \in supp\, \tau_i$, then $\tilde{V}_i(\lambda) = \tilde{V}_i(\lambda')$. We now show that if the distribution $F_H(\lambda)$ is continuous, then (i) $supp\, \tau_i$ is an interval of the form $supp\, \tau_R = [\overline{\lambda}, \lambda_{max}]$ and $supp\, \tau_L = [\lambda_{min}, \underline{\lambda}]$, (ii) $\lambda_\gamma(\overline{\lambda}) = \overline{\lambda}$ and $\lambda_\gamma(\underline{\lambda}) = \underline{\lambda}$, and (iii) λ_γ must satisfy (17) given $\overline{\lambda}$ and $\underline{\lambda}$ for any level of capture.

First, suppose that $F_H(\lambda)$ is a continuous distribution with convex support $supp\, F_H$ and let $\overline{\lambda} \equiv \max\{\lambda : \lambda_\gamma(\lambda) = \lambda, \lambda \in supp\, F_H\}$ be the highest news that sophisticated viewers interpret at face value. Since $\lambda_\gamma(\lambda) \neq \lambda$ implies that $\lambda \in supp\, \tau_R \cup \tau_L$, we must have $\min\{\lambda : \lambda \in supp\, \tau_R\} \leq \overline{\lambda}$. We show that $\min\{\lambda : \lambda \in supp\, \tau_R\} = \overline{\lambda}$. Suppose by contradiction that $\min\{\lambda : \lambda \in supp\, \tau_R\} < \overline{\lambda}$. Then the $R-$SIG obtains utility $\tilde{V}_i(\overline{\lambda}) = V_i(\overline{\lambda})$ from $\overline{\lambda}$, while any $\lambda' \in (\min\{\lambda : \lambda \in supp\, \tau_R\}, \overline{\lambda})$ gives strictly less utility as $\tilde{V}_i(\lambda') \leq V_i(\lambda') < V_i(\overline{\lambda})$. Thus, the $R-$SIG can improve by sending instead $\overline{\lambda}$, thus reaching a contradiction. A similar argument applied to the $L-$SIG implies that $supp\, \tau_L = [\lambda_{min}, \underline{\lambda}]$ and $\lambda_\gamma(\underline{\lambda}) = \underline{\lambda}$. Finally, we obtain (17) by solving for $\lambda_\gamma(\lambda)$ in

$$\begin{aligned}(1-\gamma)\, V_L(\lambda) + \gamma V_L(\lambda_\gamma(\lambda)) &= V_L(\underline{\lambda}) \quad \text{if } \lambda \leq \underline{\lambda}, \\ (1-\gamma)\, V_R(\lambda) + \gamma V_R(\lambda_\gamma(\lambda)) &= V_R(\overline{\lambda}) \quad \text{if } \lambda \geq \overline{\lambda}.\end{aligned}$$

Note that the equilibrium interpretation (17) depends on $\overline{\lambda}$ and $\underline{\lambda}$. These are pinned down in equilibrium by the condition that each SIGs probability of sending each potential lie aggregate to one. Solving for $\tau_R(\lambda)$ and $\tau_L(\lambda)$ in (35)

$$\frac{r}{1-l-r}\tau_R(\lambda) = \frac{\lambda-\lambda_\gamma(\lambda)}{\lambda_\gamma(\lambda)-1}p_{-1}(\lambda),$$
$$\frac{l}{1-l-r}\tau_L(\lambda) = \frac{\lambda_\gamma(\lambda)-\lambda}{1-\lambda_\gamma(\lambda)}p_{-1}(\lambda),$$

and integrating these expressions over the respective supports we obtain (18) and (19).

To complete the proof, we write (17) as $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ to make explicit the dependence on $(\overline{\lambda},\underline{\lambda})$ and define

$$\overline{w}(\overline{\lambda}) \equiv \int_{\overline{\lambda}}^{\infty} \frac{\lambda-\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})}{\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})-1} dF_{H,-1}(\lambda), \tag{37}$$
$$\underline{w}(\underline{\lambda}) \equiv \int_{0}^{\underline{\lambda}} \frac{\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})-\lambda}{1-\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})} dF_{H,-1}(\lambda). \tag{38}$$

First, we show that $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ is monotonic in $(\overline{\lambda},\underline{\lambda})$. Indeed, as V_R is strictly increasing (and V_L strictly decreasing), then $V_R(\overline{\lambda}) + \frac{1-\gamma}{\gamma}(V_R(\overline{\lambda}) - V_R(\lambda))$ increases in $\overline{\lambda}$ and decreases in γ for any $\lambda > \underline{\lambda}$; similarly, $V_L(\underline{\lambda}) + \frac{1-\gamma}{\gamma}(V_L(\underline{\lambda}) - V_L(\lambda))$ decreases in $\underline{\lambda}$ and increases in γ for any $\lambda < \underline{\lambda}$. Looking at (17) we conclude that, for a fixed value of λ, $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ is non-increasing in $\overline{\lambda}$ and non-decreasing in $\underline{\lambda}$.

Second, we will make use of the fact that $\frac{\lambda-x}{x-1}$ is decreasing in x for $1 < x < \lambda$, while $\frac{x-\lambda}{1-x}$ is decreasing in x for $\lambda < x < 1$. This fact and the monotonicity of $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ in $(\overline{\lambda},\underline{\lambda})$ imply that $\overline{w}(\overline{\lambda})$ in (37) is an strictly decreasing function of $\overline{\lambda}$ with $\overline{w}(\lambda_{max}) = 0$ while $\underline{w}(\underline{\lambda})$ in (38) is an strictly increasing function of $\underline{\lambda}$ with $\underline{w}(\lambda_{min}) = 0$. Furthermore, conditions (18) and (19) translate to $\overline{w}(\overline{\lambda}) = r/(1-r-l)$ and $\underline{w}(\underline{\lambda}) = l/(1-r-l)$. We can then establish uniqueness: As the left hand side of (18) is an strictly decreasing function of $\overline{\lambda}$ and the left hand side of (19) is strictly increasing function of $\underline{\lambda}$, a unique solution to (18-19) is guaranteed for every r and l.

Finally, increasing r or l raises the right hand side of (18) and (19) leading to a lower $\overline{\lambda}$ and higher $\underline{\lambda}$. Likewise, increasing γ lowers both $\overline{w}(\overline{\lambda})$ and $\underline{w}(\underline{\lambda})$, leading to a lower equilibrium $\overline{\lambda}$ and higher $\underline{\lambda}$. □

Proof of Proposition 6. Suppose that citizens anticipate a level of capture $\left(\tilde{r},\tilde{l}\right)$. The $R-$SIG's expected utility when investing r in covertly capturing the source if citizens correctly anticipate the $R-$SIG's capture effort is

$$W_R(r,l;\tilde{r},\tilde{l})\Big|_{l=\tilde{l}} = r\tilde{V}_R(\overline{\lambda}) + \tilde{l}\mathbb{E}_{\tau_L}\left[\tilde{V}_R(\lambda);p_R\right] + (1-r-l)\mathbb{E}_H\left[\tilde{V}_R(\lambda);p_R\right] - C_R(r).$$

with

$$\mathbb{E}_H\left[\tilde{V}_R(\lambda);p_R\right] = \overline{F}_H(\overline{\lambda};p_R)V_R(\overline{\lambda}) + \int_{\lambda_{min}}^{\overline{\lambda}}\left(\left(1-\gamma\right)V_R(\lambda) + \gamma V_R(\lambda_\gamma(\lambda))\right)dF_H(\lambda;p_R). \tag{39}$$

Therefore, the $R-$SIG 's marginal gain from covertly increasing media capture $B_R(\tilde{r},\tilde{l}) \equiv \frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r}\Big|_{l=\tilde{l}}$ is

$$\begin{aligned} B^R(\tilde{r},\tilde{l}) &= V_R(\overline{\lambda}) - \mathbb{E}_H\left[V_R(\lambda);p_R\right] \\ &= \int_{\lambda_{min}}^{\overline{\lambda}}\left(V_R(\overline{\lambda}) - V_R(\lambda)\right)dF_H(\lambda;p_R) \qquad (40) \\ &\quad -(1-\gamma)\int_{\lambda_{min}}^{\underline{\lambda}}\left(V_R(\lambda_\gamma(\lambda)) - V_R(\lambda)\right)dF_H(\lambda;p_R). \qquad (41) \end{aligned}$$

By increasing capture efforts, the $R-$SIG obtains $V_R(\overline{\lambda})$ instead of the utility derived from an honest coverage $\mathbb{E}_H\left[V_R(\lambda);p_R\right]$.Thus, the $R-$SIG gains $V_R(\overline{\lambda}) - V_R(\lambda)$ whenever $\lambda \leq \overline{\lambda}$ and all viewers (including sophisticated ones) interpret the message at face value –this is (40)– except when $\lambda \leq \underline{\lambda}$ and sophisticated viewers discount the news –this is (41).

We now show that $\partial B^R(\tilde{r},\tilde{l})/\partial\tilde{l} \leq 0$ so the $R-$SIG's incentives to capture decrease with the anticipated level of capture of the $L-$SIG. First, part 3 of Proposition 5 shows that $\overline{\lambda}$ decreases with l, so (40) decreases with $\tilde{l}$. Moreover, part 3 of Proposition 5 also shows that increasing l, (a) increases $\lambda_\gamma(\lambda)$ for $\lambda \leq \underline{\lambda}$, and (b) increases $\underline{\lambda}$. Both effects raise the value of the integral in (41), thus decreasing (41). Therefore, increasing $\tilde{l}$ lowers $B^R(\tilde{r},\tilde{l})$. A similar analysis applied to capture by the $L-$SIG shows that $\partial B^L(\tilde{r},\tilde{l})/\partial\tilde{r} \leq 0$. □

Proposition 11. ***(Existence of pure-strategy capture equilibria)*** *Consider a market with*

n different information sources. SIGs have (i) continuous utilities $v_i(\mu)$, $\mu \in [0,1], i \in \{R, L\}$; (ii) continuous and convex costs of capture $C_R(r)$ and $C_R(l)$ with $r \in \Pi_{j=1}^n X_R^j$, and $l \in \Pi_{j=1}^n X_L^j$; and (iii) for each source $j \in \{1, ..., n\}$, the probability of state $S^j = i$, $\pi_i^j(r_j, l_j)$, is continuous and concave in r_j and concave in l_j with $\pi_H^j(r_j, l_j) > 0$ for $r_j \in X_R^j, l_j \in X_L^j$. Then, there is an equilibrium with pure-strategies capture efforts (r^*, l^*).

Proof. Suppose that the $R-SIG$ selects $r = (r_j)_{j=1}^n$; the $L-SIG$ selects $l = (l_j)_{j=1}^n$; and citizens have an assessment of $SIGs'$ capture strategies $(\tilde{r}, \tilde{l})$ and an assessment of reporting strategies $(\tilde{\tau}_R, \tilde{\tau}_L)$ that is consistent with Proposition 1 given $(\tilde{r}, \tilde{l})$. Then, the payoffs to each SIG are,

$$W_R(r, l; \tilde{r}, \tilde{l}) = \sum_{j=1}^n \left(\pi_R^j(r_j, l_j) V_R^j(\overline{\lambda}_j) + \pi_L^j(r_j, l_j) V_R^j(\underline{\lambda}_j) + \pi_H^j(r_j, l_j) \mathbb{E}_H^j \left[V_R^j(\lambda); p_R\right]\right) - C_R(r) \quad (42)$$

$$W_L(r, l; \tilde{r}, \tilde{l}) = \sum_{j=1}^n \left(\pi_R^j(r_j, l_j) V_L^j(\overline{\lambda}_j) + \pi_L^j(r_j, l_j) V_L^j(\underline{\lambda}_j) + \pi_H^j(r_j, l_j) \mathbb{E}_H^j \left[V_L^j(\lambda); p_L\right]\right) - C_L(l) \quad (43)$$

with $\overline{\lambda}_j$ and $\underline{\lambda}_j$ satisfying (4) and (5) with $r_j = \tilde{r}_j$, $l_j = \tilde{l}_j$, and $V_i^j(\lambda)$, $i \in R, L$, given by (8) with $F_p = F_p^j$. We can then define the $i-SIG$'s best-response correspondence given citizens' assessment $(\tilde{r}, \tilde{l})$,

$$\tilde{\Psi}_R(l; \tilde{r}, \tilde{l}) \equiv \{r : W_R(r, l; \tilde{r}, \tilde{l}) \geq W_R(r', l; \tilde{r}, \tilde{l}), r' \in \Pi_{j=1}^n X_R^j\}, \quad (44)$$

$$\tilde{\Psi}_L(r; \tilde{r}, \tilde{l}) \equiv \{l : W_L(r, l; \tilde{r}, \tilde{l}) \geq W_L(r, l'; \tilde{r}, \tilde{l}), l' \in \Pi_{j=1}^n X_L^j\}, \quad (45)$$

and the belief-consistent best-response correspondence

$$\tilde{\Psi}(r, l) \equiv \{\tilde{\Psi}_R(l; r, l), \tilde{\Psi}_L(r; r, l)\}.$$

Note that (r^*, l^*) is a pure-strategy-in-capture-efforts equilibrium if and only if $(r^*, l^*) \in \tilde{\Psi}(r^*, l^*)$. We will apply standard existence results in continuous games with quasiconcave payoffs (see, Debreu (1952), Glicksberg (1952) and Fan (1952)) to show that $\tilde{\Psi}$ has a fixed point.

First, we establish that $W_i(r, l; \tilde{r}, \tilde{l})$ is continuous at each $(r, l; \tilde{r}, \tilde{l})$, and that $W_R(W_L)$ is concave in $r(l)$. For continuity, it suffices to show that $V_i^j(\overline{\lambda}_j), V_i^j(\underline{\lambda}_j)$ and $\mathbb{E}_H^j \left[V_i^j(\lambda); p_i\right]$ are continuous.

Define the functions

$$\overline{Q}_j(\lambda) \equiv \frac{\int_\lambda^\infty \overline{F}^j_{H,-1}(\lambda')d\lambda'}{\lambda - 1}; \underline{Q}_j(\lambda) \equiv \frac{\int_0^\lambda F^j_{H,-1}(\lambda')d\lambda'}{1-\lambda}.$$

Note that $\overline{Q}_j(\lambda) \in \mathbb{R}_{>0}$ is continuous and strictly decreasing for $\lambda > 1$, while $\underline{Q}_j(\lambda) \in \mathbb{R}_{>0}$ is continuous and strictly increasing for $0 \leq \lambda < 1$, thus both possessing a continuous inverse in $\mathbb{R}_{>0}$. The equilibrium thresholds (4-5) imply

$$V_i^j(\overline{\lambda}_j) = V_i^j(\overline{Q}_j^{-1}(\frac{\pi_R^j(r_j, l_j)}{\pi_H^j(r_j, l_j)})),$$
$$V_i^j(\underline{\lambda}_j) = V_i^j(\underline{Q}_j^{-1}(\frac{\pi_L^j(r_j, l_j)}{\pi_H^j(r_j, l_j)})),$$

which are continuous as the composition of continuous functions –as $\pi_H^j(r_j, l_j) > 0$ for $r_j \in X_R^j, l_j \in X_L^j$. Concavity of $W_R(W_L)$ in $r(l)$ follows immediately from concavity of $\pi_i^j(r_j, l_j)$ with respect to $r_j(l_j)$ and convexity of $C_R(r)$ and $C_L(l)$.

As X_R^j and X_L^j are compact and convex for each $j = 1, .., n$, continuity of W_R and W_L implies that $\tilde{\Psi}_R(l; \tilde{r}, \tilde{l})$ and $\tilde{\Psi}_L(r; \tilde{r}, \tilde{l})$ are upper-hemicontinuous and concavity of W_R and W_L imply that they are convex-valued. Upper-hemicontinuity is preserved when restricting attention to the subset $\{(l; \tilde{r}, \tilde{l}) : l = \tilde{l}\}$ and $\{(r; \tilde{r}, \tilde{l}) : r = \tilde{r}\}$. Therefore, $\tilde{\Psi}(r, l)$ is non-empty, convex-valued and upper-hemicontinuous and Kakutani's fixed-point theorem guarantees the existence of a fixed point. □

Proof of Proposition 7. For any equilibrium level of capture $(r^*, l^*) = ((r_j^*)_{j=1}^n, (l_j^*)_{j=1}^n)$, let $\hat{r} = \sum_{j=1}^n \beta_j^R r_j^*$ and $\hat{l} = \sum_{j=1}^n \beta_j^L l_j^*$. Applying to an oligopoly market the equilibrium conditions given citizens' consistent beliefs (12-15) requires that (i) for each source in which $r_j^* > 0$ ($l_j^* > 0$) we must have

$$B_j^R(r_j^*, l_j^*) = \beta_j^R C_R'(\hat{r}) \; (B_j^L(r_j^*, l_j^*) = \beta_j^L C_L'(\hat{l})) \tag{46}$$

and (ii) for each source for which $r_j^* = 0$ ($l_j^* = 0$) we must have

$$B_j^R(r_j^*, l_j^*) \leq \beta_j^R C_R'(\hat{r})(B_j^L(r_j^*, l_j^*) \leq \beta_j^L C_L'(\hat{l})).$$

Consider an equilibrium capture profile (r^*, l^*) and suppose that either $r_n^* > 0$ or $l_n^* > 0$. By contradiction, suppose that $r_j^* = l_j^* > 0$ for some $j \in \{1, .., n-1\}$. Then, symmetry of costs and (46) requires $C_R'(\hat{r}) = C_L'(\hat{l})$, and strict convexity of C_i implies that $\hat{r} = \hat{l}$. It also implies that if source $j' \in \{1, .., n-1\}$ is captured –i.e., if $r_{j'}^* > 0$ or $l_{j'}^* > 0$– then we must have $r_{j'}^* = l_{j'}^*$ –a consequence of (46) and the assumption that symmetric returns $B_{j'}^R(r_{j'}^*, l_{j'}^*) = B_{j'}^L(r_{j'}^*, l_{j'}^*)$ imply equal capture levels $r_{j'}^* = l_{j'}^*$. Therefore, for every $j' \in \{1, .., n-1\}$ we must have $r_{j'}^* = l_{j'}^*$. Finally, since $\beta_n^R = \beta_n^L$ we must also have that $r_n^* = \left(\hat{r} - \sum_{j=1}^{n-1} \beta_j^R r_j^*\right) / \beta_n^R = \left(\hat{l} - \sum_{j=1}^{n-1} \beta_j^L l_j^*\right) / \beta_n^L = l_n^*$. But then, the optimality condition (46) cannot be satisfied for source n as if $r_n^* = l_n^*$, then

$$\beta_R^n C_R'(\hat{r}) = B_R^n(r_n^*, l_n^*) \neq B_L^n(r_n^*, l_n^*) = \beta_L^n C_L'(\hat{l})$$

but symmetric costs implies $\beta_R^n C_R'(\hat{r}) = \beta_L^n C_L'(\hat{l})$, thus reaching a contradiction. □

The proof of Proposition 8 will make use of the following two lemma. To state these results, define $\Psi_R(l)$ and $\Psi_L(r)$ as the best response correspondence by the $R-$SIG and $L-$SIG when citizens correctly anticipate both SIGs capture efforts; in other words,[41]

$$r \in \Psi_R(l) \iff r \in \tilde{\Psi}_R(l; r, l); l \in \Psi_L(r) \iff l \in \tilde{\Psi}_R(r; r, l), \tag{47}$$

with $\tilde{\Psi}_i$ the best response correspondence defined in (44-45). To explicitly characterize, say, $\Psi_R(l)$, let $h_i(c) = (C_i')^{-1}(c)$ be the inverse of the marginal cost for $i \in \{R, L\}$. Given $l = (l_1, l_2)$, suppose, for example, that $\left(1/\beta_1^R\right) B_1^R(0, l_1) > \left(1/\beta_2^R\right) B_2^R(0, l_2)$. Then, the conditions defining any $r \in \Psi_R(l)$ are

$$\beta_1^R r_1 = h_R(\left(1/\beta_1^R\right) B_1^R(r_1, l_1)) > h_R(\left(1/\beta_2^R\right) B_2^R(0, l_2)), if\ r_2 = 0, \tag{48}$$

$$\beta_1^R r_1 + \beta_2^R r_2 = h_R(\left(1/\beta_1^R\right) B_1^R(r_1, l_1)) = h_R(\left(1/\beta_2^R\right) B_2^R(r_2, l_2)), if\ r_2 > 0. \tag{49}$$

The first lemma shows that each SIG's best response to a rotation in the strategy of the other SIG –i.e., increasing one capture effort but lowering the other– is itself a rotation of opposite sign.

[41]So $\Psi_R(l)$ ($\Psi_L(l)$) is the set of fixed points of $\tilde{\Psi}_R(l; \cdot, l)$ ($\tilde{\Psi}_L(r; r, \cdot)$).

The second lemma provides simple comparative statics on Ψ_R and Ψ_L with changes in the reach of a source or the cost of capturing that source.

Lemma 5. ***(Rotations are best-responses)*** *With $\Psi_R(l)$ and $\Psi_L(r)$ defined by (47), let $r = (r_1, r_2)$, $l = (l_1, l_2), r' = (r'_1, r'_2)$, and $l' = (l'_1, l'_2)$.*

i-If $l'_1 \geq (\leq) l_1$, $l'_2 \leq (\geq) l_2$, $r \in \Psi_R(l)$ and $r' \in \Psi_R(l')$, then $r'_1 \leq (\geq) r_1$ and $r'_2 \geq (\leq) r_2$.

ii-If $r'_1 \geq (\leq) r_1$, $r'_2 \leq (\geq) r_2$, $l \in \Psi_R(r)$ and $l' \in \Psi_R(r')$, then $l'_1 \leq (\geq) l_1$ and $l'_2 \geq (\leq) l_2$.

Proof. We prove this lemma for case (i) as case (ii) follows along similar arguments, and only for the counterclockwise rotation ($l'_1 \leq l_1, l'_2 \geq l_2$) as the clockwise case follows similarly.

Select an $r \in \Psi_R(l)$ and suppose that $l'_1 \leq l_1$ and $l'_2 \geq l_2$, we will show that for any $r' \in \Psi_R(l')$, we have $r'_1 \geq r_1, r'_2 \leq r_2$. First, suppose by way of contradiction that $r'_1 < r_1$. As $B^R_j(r_j, l_j)$ is non-increasing in r_j and l_j –see (9-10) and Proposition 2– and $l'_1 \leq l_1$, then we must have $B^R_1(r'_1, l'_1) \geq B^R_1(r_1, l_1)$. If $r'_1 = 0$, then (49) implies that

$$\beta^R_2 r'_2 = h_R(\left(1/\beta^R_2\right) B^R_2(r'_2, l'_2)) > h_R(\left(1/\beta^R_1\right) B^R_1(0, l'_1)) \geq h_R(\left(1/\beta^R_1\right) B^R_1(r_1, l_1)) = \beta^R_1 r_1 + \beta^R_2 r_2, \tag{50}$$

which implies that $r'_2 > r_2$. Since $r'_2 > r_2$ and $l'_2 \geq l_2$ imply that $B^R_2(r'_2, l'_2) \leq B^R_2(r_2, l_2)$, then we must have

$$\beta^R_2 r'_2 = h_R(\left(1/\beta^R_2\right) B^R_2(r'_2, l'_2)) \leq h_R(\left(1/\beta^R_2\right) B^R_2(r_2, l_2)) = \beta^R_1 r_1 + \beta^R_2 r_2,$$

but this expression is incompatible with (50), reaching a contradiction.

If instead $r'_1 > 0$, then r'_1 satisfies (49) and $B^R_2(r'_2, l'_2) = \left(\beta^R_2/\beta^R_1\right) B^R_1(r'_1, l'_1) \leq \left(\beta^R_2/\beta^R_1\right) B^R_1(r_1, l_1) = B^R_2(r_2, l_2)$ implying that $r'_2 \leq r_2$ as $l'_2 \geq l_2$. But then, we reach a contradiction as r' cannot be optimal since

$$\beta^R_1 r'_1 + \beta^R_2 r'_2 < \beta^R_1 r_1 + \beta^R_2 r_2 = h_R(\left(1/\beta^R_1\right) B^R_1(r_1, l_1)) \leq h_R(\left(1/\beta^R_1\right) B^R_1(r'_1, l'_1)).$$

Second, suppose that $r'_2 > r_2$. Then $B^R_2(r'_2, l'_2) \leq B^R_1(r_1, l_1)$ as $l'_2 \geq l_2$. Since $r'_2 > 0$, it must satisfy (49), so we must have $B^R_1(r'_1, l'_1) \leq B^R_1(r_1, l_1)$ implying that $r'_1 \geq r_1$ as $l'_1 \leq l_1$. But then, we reach a contradiction as r' cannot be optimal since

$$\beta_1^R r_1' + \beta_2^R r_2' > \beta_1^R r_1 + \beta_2^R r_2 = h_R(\left(1/\beta_1^R\right) B_1^R(r_1, l_1)) \geq h_R(\left(1/\beta_1^R\right) B_1^R(r_1', l_1)).$$

□

Lemma 6. ***(Comparative Statics-Direct Effect)*** *Consider an equilibrium (r^*, l^*) and suppose that either (a) both SIGs want to fire-up-the-base (demobilize the opposition) and $F_1^p(p)$ increases (decreases) in the FOSD sense, or (b) the $R-SIG'$s cost parameters change according to $\tilde{\beta}_1^R = \beta_1^R - \delta_1$ and $\tilde{\beta}_2^R = \beta_2^R + \delta_2$, $\delta_1, \delta_2 > 0$, with $\delta_2/\delta_1 = r_1^*/r_2^*$. Let $\Psi_R^\delta(l)$ and $\Psi_L^\delta(r)$ be the best response correspondences after the change in parameters. Then for any $\overline{r} = (\overline{r}_1, \overline{r}_2) \in \Psi_R^\delta(l^*)$ and $\bar{l} = (\bar{l}_1, \bar{l}_2) \in \Psi_L^\delta(r^*)$, we have $\overline{r}_1 \geq r_1^*$, $\overline{r}_2 \leq r_2^*$, $\bar{l}_1 \leq l_1^*$, and $\bar{l}_2 \geq l_2^*$.*

Proof. Consider first case (a) with $F_1^p(p)$ increasing in the FOSD sense, and let $B_1^{j,\delta}(r_1, l_1)$, $j \in \{R, L\}$, be the marginal gain after the change in source 1's reach. As both SIGs want to fire-up-their-base, then we must have that $B_1^{R,\delta}(r_1, l_1) \geq B_1^R(r_1, l_1)$ and $B_1^{L,\delta}(r_1, l_1) \leq B_1^L(r_1, l_1)$. But then, conditions (48-49) imply that for any $r \in \Psi_R(l)$ and $r' \in \Psi_R^\delta(l)$, we must have $r_1' \geq r_1$ and $r_2' \leq r_2$ –see proof of Proposition 4. Similarly for the $L-$SIG, we must have that for any $l \in \Psi_L(r)$ and $l' \in \Psi_L^\delta(r)$, $l_1' \leq l_1$ and $l_2' \geq l_2$.

Consider now case (b) with an initial equilibrium (r^*, l^*) and a change $\tilde{\beta}_1^R = \beta_1^R - \delta_1$ and $\tilde{\beta}_2^R = \beta_2^R + \delta_2$. The condition $\delta_2/\delta_1 = r_1^*/r_2^*$ implies that the cost of capture under r^* remains invariant since

$$\tilde{\beta}_1^R r_1^* + \tilde{\beta}^R r_2^* = \beta_1^R r_1^* + \beta_2^R r_2^*. \tag{51}$$

Suppose that $r_1^*, r_2^* > 0$, so that (49) holds for r^* and let $\overline{r} \in \Psi_R^\delta(l^*)$. We prove the claim by contradiction. To derive a contradiction, suppose that $0 < \overline{r}_1 < r_1^*$. Then $B_1^R(\overline{r}_1, l_1^*) \geq B_1^R(r_1^*, l_1^*)$.

The optimality condition (49) then implies

$$\begin{aligned}
\tilde{\beta}_1^R C_R'\left(\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2\right) &\geq \beta_1^R C_R'\left(\beta_1^R r_1^* + \beta_2^R r_2^*\right) \\
\implies \tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2 &\geq \beta_1^R r_1^* + \beta_2^R r_2^* \\
\implies \tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2 &\geq \tilde{\beta} r_1^* + \tilde{\beta}_2^R r_2^* \\
\implies \tilde{\beta}_2^R \overline{r}_2 &\geq \underbrace{\tilde{\beta}_1^R (r_1^* - \overline{r}_1)}_{>0} + \beta_2^R r_2^* \\
\implies \overline{r}_2 &> r_2^*,
\end{aligned}$$

where the first implication follows from convexity of C_R and $\tilde{\beta}_1^R < \beta_1^R$, and the second implication uses (51). As $\overline{r}_2 > r_2^*$, then we must have $B_2^R(\overline{r}_2, l_2^*) \leq B_1^R(r_2^*, l_2^*)$. But this leads to a contradiction as $\tilde{\beta}_2^R > \beta_2^R$ and $\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2 \geq \beta_1^R r_1^* + \beta_2^R r_2^*$ imply that

$$\tilde{\beta}_2^R C_R'\left(\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2\right) > \beta_2^R C_R'\left(\beta_1^R r_1^* + \beta_2^R r_2^*\right) = B_2^R(r_2^*, l_2^*) \geq B_2^R(\overline{r}_2, l_2^*),$$

and $\overline{r}_2$ cannot be optimal.

Similarly, if we suppose that $\overline{r}_2 > r_2^*$ then $B_2^R(\overline{r}_2, l_2^*) \leq B_2^R(r_2^*, l_2^*)$, and the condition (49) implies that

$$\begin{aligned}
\tilde{\beta}_2^R C_R'\left(\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2\right) &\leq \beta_2^R C_R'\left(\beta_1^R r_1^* + \beta_2^R r_2^*\right) \\
\implies \tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2 &\leq \beta_1^R r_1^* + \beta_2^R r_2^* \\
\implies \tilde{\beta}_1^R \overline{r}_1 &\leq \underbrace{\tilde{\beta}_2^R (r_2^* - \overline{r}_2)}_{<0} + \beta_1^R r_1^* \\
\implies \overline{r}_1 &< r_1^*.
\end{aligned}$$

As $\overline{r}_1 < r_1^*$, then we must have $B_1^R(\overline{r}_1, l_1^*) \geq B_1^R(r_1^*, l_1^*)$. But this leads to a contradiction as $\tilde{\beta}_1^R < \beta_1^R$ and $\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2 \leq \beta_1^R r_1^* + \beta_2^R r_2^*$ imply that

$$\tilde{\beta}_1^R C_R'\left(\tilde{\beta}_1^R \overline{r}_1 + \tilde{\beta}_2^R \overline{r}_2\right) < \beta_1^R C_R'\left(\beta_1^R r_1^* + \beta_2^R r_2^*\right) = B_1^R(r_1^*, l_1^*) \leq B_1^R(\overline{r}_1, l_1^*),$$

and $\overline{r}_1$ cannot be optimal. $\square$

Proof of Proposition 8. Given an equilibrium (r^*, l^*), define the set of counter-rotations

$$T^*(r^*, l^*) \equiv \{(r, l) : r_1 \geq r_1^*, r_2 \leq r_2^*, l_1 \leq l_1^*, l_2 \geq l_2^*\}$$

which is a non-empty, compact and convex set.

Let Ψ_i^δ be the best response correspondence defined in (47) after the change in source parameters (either change in $F_1(p)$, or the change in cost parameters). Lemma 6 establishes that $(\Psi_R^\delta(l^*), \Psi_L^\delta(r^*)) \subset T^*(r^*, l^*)$. Furthermore, fixing $\overline{r} \in \Psi_R^\delta(l^*)$ and $\bar{l} \in \Psi_L^\delta(r^*)$, Lemma 5 guarantees that for any $(r', l') \in T^*(r^*, l^*)$ and $(r'', r'') \in (\Psi_R^\delta(r'), \Psi_L^\delta(l'))$ we must have $r_1'' \geq \overline{r}_1, r_2'' \leq \overline{r}_2, l_1'' \leq \bar{l}_1$, and $l_2'' \geq \bar{l}_2$ so that $(\Psi_R^\delta(r'), \Psi_L^\delta(l')) \subset T^*(r^*, l^*)$. Finally, continuity of Ψ_i and Ψ_i^δ follows from continuity of the best response correspondence $\tilde{\Psi}_R(l; \tilde{r}, \tilde{l})$ and $\tilde{\Psi}_L(r; \tilde{r}, \tilde{l})$ in (44-45). Therefore, the best response correspondence satisfies the conditions of Kakutani's fixed-point theorem so that a fixed point exists that is a counter-rotation of SIGs strategies. The proof is then complete once we observe that if $r_1^*/r_2^* > l_1^*/l_2^*$, then any $\tilde{r}$ and $\tilde{l}$ satisfying

$$\tilde{r}_1 \geq r_1^*, \tilde{r}_2 \leq r_2^*, \tilde{l}_1 \leq l_1^*, \tilde{l}_2 \geq l_2^*$$

must necessarily satisfy $\mathscr{P}_G(\tilde{r}, \tilde{l}) \geq \mathscr{P}_G(r^*, l^*)$ and $\mathscr{P}_I(\tilde{r}, \tilde{l}) \geq \mathscr{P}_I(r^*, l^*)$ $\square$

Proof of Proposition 9. Recall that for each viewer with prior p, $\lambda_{crit}(p) = (1-p)/p$ is the minimum informational content of a message that would lead her to act. We first derive the instrumental value of a $p-$viewer from a captured source, and then study how the difference in instrumental values between sources 1 and 2 varies with p.

Let $F_\lambda^j(\lambda, p)$, $j \in \{1, 2\}$, be the perceived equilibrium distribution of likelihood ratios of source j by a $p-$viewer–see (7)–and $F_\mu^j(\mu, p) \equiv F_\lambda^j(\frac{\mu}{1-\mu}\frac{1-p}{p}, p)$ be the distribution of posterior beliefs after she consumes the news of source j. The instrumental value of that viewer, $W_I^j(p)$, if $p > 1/2$ is

$$W_I^j(p) \equiv \int_0^{1/2} [(1/2)(1-\mu) - (1/2)\mu] dF_\mu^j(\mu, p) = \int_0^{1/2} [1/2 - \mu] dF_\mu^j(\mu, p) = \int_0^{1/2} F_\mu^j(\mu, p) d\mu$$
$$= \int_0^{\lambda_{crit}(p)} F_\lambda^j(\lambda, p) \frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda$$

where we made the change of variables $\lambda = \frac{\mu}{1-\mu}\frac{1-p}{p}$ to obtain the last term. This follows as the viewer will change her decision from $a = 1$ to $a = 0$ only after observing a message that leads her to a posterior belief $\mu \leq 1/2$ –i.e., a message with $\lambda \leq \lambda_{crit}(p)$. Equivalently, if $p < 1/2$ we have

$$W_I^j(p) \equiv \int_\alpha^1 [(1/2)\mu - (1/2)(1-\mu)] dF_\mu^j(\mu, p) = \int_{1/2}^1 [\mu - (1/2)] dF_\mu^j(\mu, p) = \int_{1/2}^1 \overline{F}_\mu^j(\mu, p) dp$$
$$= \int_{\lambda_{crit}(p)}^1 \overline{F}_\lambda^j(\lambda, p) \frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda.$$

Let $\Delta_F(\lambda, p) = F_\lambda^1(\lambda, p) - F_\lambda^2(\lambda, p)$ be the difference in the equilibrium distribution of likelihood ratios between source 1 and source 2, and $\Delta_W(p) \equiv W_I^1(p) - W_I^2(p)$ be the difference in instrumental value between both sources. Then, the $p-$viewer with $p > 1/2$ will consume source 1 whenever

$$\Delta_W(p) = \int_0^{\lambda_{crit}(p)} \Delta_F(\lambda, p) \frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda \geq 0$$

and will consume source 2 otherwise. Similarly, a $p-$viewer with $p < 1/2$ will consume source 1 if

$$\Delta_W(p) = \int_{\lambda_{crit}(p)}^1 \left(-\Delta_F(\lambda, p)\right) \frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda \geq 0.$$

Suppose $r^1 \geq l^1$, $l^2 \geq r^2$, and (20) holds so capture levels are not too dissimilar. We now show that we must have

$$\overline{\lambda}_1 < \overline{\lambda}_2 \text{ and } \underline{\lambda}_2 > \underline{\lambda}_1; \tag{52}$$

that is the highest equilibrium likelihood ratio is smaller in the right-dominated media while the lowest one is higher in the left-dominated media. Note first that (20) implies that $\frac{r^1}{1-(r^1+l^1)} > \frac{r^2}{1-(r^2+l^2)}$ and $\frac{l^2}{1-(r^2+l^2)} > \frac{l^1}{1-(r^1+l^1)}$, i.e., the likelihood that the message is sent by the $R-$SIG rather than the honest sender is higher in media 1, while the likelihood that the message is sent by the $L-$SIG rather than the honest sender is higher in media 2. As $F_H^1 = F_H^2 \, (= F_H)$ so that $F_{H,-1}^1(\lambda) = F_{H,-1}^2(\lambda)$, (4) and (5) imply (52).

Given symmetry of the channel and the relation between the maximum and minimum likelihood ratios (52), we can write $\Delta_F(\lambda, p)$ as

$$\Delta_F(\lambda,p) = \begin{cases} 0 & if \quad \lambda < \underline{\lambda}_1 \\ (1-(r^1+l^1))\, F_H(\lambda,p) + l^1 & if \quad \underline{\lambda}_1 \leq \lambda < \underline{\lambda}_2 \\ ((r^2+l^2)-(r^1+l^1))\, F_H(\lambda,p) - (l^2-l^1) & if \quad \underline{\lambda}_2 \leq \lambda < \overline{\lambda}_1 \\ 1-(1-(r^2+l^2))\, F_H(\lambda,p) - l^2 & if \quad \overline{\lambda}_1 \leq \lambda < \overline{\lambda}_2 \\ 0 & if \quad \lambda \geq \overline{\lambda}_2 \end{cases}$$

Note that $\Delta_F(\lambda, p) \geq 0$ if $\lambda < \underline{\lambda}_2$ or if $\lambda \geq \overline{\lambda}_1$. Therefore, $\Delta_W(p) \geq 0$ if $\lambda_{crit}(p) < \underline{\lambda}_2$ –i.e., if $p > 1/(1+\underline{\lambda}_2)$– but $\Delta_W(p) \leq 0$ if $\lambda_{crit}(p) > \overline{\lambda}_1$ –i.e., if $p < 1/(1+\underline{\lambda}_1)$. This proves part i.

Suppose, in addition, that $r^1 + l^1 = r^2 + l^2$. Then $\Delta_F(\lambda, p) = -(l^2 - l^1)$ for $\underline{\lambda}_2 \leq \lambda < \overline{\lambda}_1$ which does not change sign. We now show that this implies that $\Delta_W(p)$ is strictly single-crossing in p, which proves part ii. Note that, for $p > 1/2$, $\Delta_W(p)$ must be single-crossing, from positive to negative, as $\Delta_F(\lambda, p)$ changes sign at most once from positive to negative (i.e., at $p = 1/(1+\underline{\lambda}_2)$ if $l^2 > l^1$). Likewise, for $p < 1/2$, $\Delta_W(p)$ must be single-crossing, from positive to negative as $\Delta_F(\lambda, p)$ changes sign at most once, from negative to positive –i.e., at $p = 1/(1+\overline{\lambda}_1)$ if $l^2 > l^1$. Continuity of $\Delta_W(p)$ at $p = 1/2$ implies that the sign of $\Delta_W(p)$ must not change for either $\lambda_{crit} < 1$ or $\lambda_{crit} > 1$, proving that $\Delta_W(p)$ is single-crossing. □

Proof of Proposition 10. The functional forms of v_R and v_L guarantee that both SIGs want to fire up their base –see Lemma 4. The equilibrium thresholds $\bar{\lambda}_1$ and $\underline{\lambda}_2$ ensure that the instrumental value of source 1 relative to source 2, $\Delta_W(p) = W_I^1(p) - W_I^2(p)$, is positive for $p \geq 1/(1+\underline{\epsilon}) > 1/(1+\underline{\lambda}_2)$ and negative for $p \leq 1/(1+\bar{\epsilon}) < 1/(1+\bar{\lambda}_1)$ –see proof of Proposition 9. As the distribution of viewers is such that $p \geq 1/(1+\underline{\epsilon})$ for any citizen in A and $p \leq 1/(1+\bar{\epsilon})$ for any citizen in B, then all citizens in A (B) prefer to consume source 1 (2) if they were to sort according to instrumental value. Note that since $\bar{\lambda}_1$and $\underline{\lambda}_2$ vary smoothly with ρ, marginally increasing ρ

will respect these inequalities, so that any citizen in A (B) that sorts according to instrumental value will consume source 1 (2).

Let $F_p^A(p) = \Pr[p' \leq p|A]$ and $F_p^B(p) = \Pr[p' \leq p|B]$ be the distribution of priors of citizens in groups A and B. Then, the reach of sources 1 and 2 are

$$F_p^1(p) = \frac{1+\rho}{2} F_p^A(p) + \frac{1-\rho}{2} F_p^B(p) \tag{53}$$

$$F_p^2(p) = \frac{1-\rho}{2} F_p^A(p) + \frac{1+\rho}{2} F_p^B(p). \tag{54}$$

This follows as viewers that do not sort according to instrumental value are equally likely to choose either source, while those that sort according to instrumental value do not vary the source they patronize after the increase in ρ. Note also that, as the sizes of both groups are the same, the total mass of viewers in both sources is the same. As viewers sorting preferences did not change, then increasing ρ leads to a FOSD increase in F_p^1 in (53) and a FOSD decrease in F_p^2 in (54).

As both SIGs want to fire up their base, this increases the R-SIG incentives to capture media 1, and lowers its incentives to capture media 2, while it decreases the L-SIG incentives to capture media 2, and lowers its incentives to capture media 1. Proposition 8 shows that this leads to a new equilibrium where the equilibrium level of $R-$capture increases in media 1, and decreases in media 2, while L-capture increases in media 2 and decreases in media 1, thus increasing both measures of polarization $\mathscr{P}_G(r,l)$ and $\mathscr{P}_I(r,l)$. $\square$