Price Competition, Information Acquisition, and Product Differentiation Perception*

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Abstract

We consider the equilibrium interplay between sellers' price competition and consumers' perception of product differentiation. We analyze a situation where, before trading, consumers acquire information at a cost about their preferences between sellers' differentiated products. The incentive for information acquisition depends on the average value of products, the objective product differentiation, and their beliefs about sellers' prices. The acquired information shapes consumers' perceived product differentiation and sellers' equilibrium prices. We characterize the unique symmetric equilibrium and study comparative statics with respect to consumer information acquisition cost and sensitivity to product differentiation. We then apply our model to platform design.

Key Words: Duopoly Competition, Information Acquisition, Pricing, Product Differentiation, Perception

JEL Classification: D82, D83

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1 Introduction

It is often the case where a consumer has not engaged in making a purchase in the past or is an infrequent purchaser and initially has little knowledge of what product features or services would be the best match for their tastes. For example, when renovating a kitchen a consumer will need to choose what cabinets, counter tops, flooring, stove tops, refrigerator that they will need. There are various styles that a consumer can choose from and the consumer will want to learn which style is the best satisfies their preferences before getting quotes from contractors who may have very different styles. This will enable the consumer to make a more informed decision about which, if any, bid to accept. More generally, this is the case when making a purchase of most durable goods such as items for homes (HVAC units and major appliances), consumer electronics (computers, printers, and smart phones) and transportation (automobiles, bikes, and scooters). These are products where typically consumer's knowledge of the current attributes are minimal, while at the same time there are multiple potential sellers with competing versions of the product. For each of these products, the consumer will typically try to learn what relevant attributes these products have and what is his preference for each attribute before contacting sellers for price quotes.

In this paper, we analyze a consumer in a horizontally differentiated setting where the consumer is initially uncertain which product is the best match for her. What is novel about our approach is that a consumer perceived product differentiation is endogenously determined by expected prices, while the prices are determined by how knowledgeable consumers are about the *perceived* level of product differentiation concerning what product is the best match for her. Formally, we allow consumers to choose the level of an experiment, at a cost, generating a signal of which product is the best match. The sellers, knowing that a consumer is conducting an experiment, choose their prices. A consumer then learns the result of the experiment and chooses which, if any, purchase to make.

Our approach allows for the perception of how differentiated products are for consumers to be determined endogenously in equilibrium with how prices are formed. We think that this has a great deal of intuitive appeal. If a consumer has very little information about products, then she will think of the products as being very similar ex-ante. This will induce stiff price competition among firms. On the other hand, when she gathers a great deal about the products via running an experiment to identify which product is best for her, then the products become more differentiated. This softens price competition between firms. This idea cannot easily be implemented in the standard search models, whether the products are all viewed to have identical attributes and consumer just cares about finding the lowest price (Stahl, 1989), or if the goods are heterogeneous but of fixed quality perceptions for a consumer (Wolinsky, 1986; Anderson and Renault, 1999). We discuss the literature more fully below.

In our base model, we have two sellers and a buyer. The buyer is uncertain about which of the two sellers' products would be a better match for her tastes. Simultaneously, the sellers choose their price and the buyer runs a costly experiment to see which product is best fit for her. The buyer then observes the outcome of the experiment and the prices and decides which, if either, seller to purchase from.

We first characterize the equilibrium. We find that the symmetric equilibrium is unique and sellers play, depending on parameters, one of two classes of mixed strategy equilibria. In one class, the density is positive for every price in a closed set with no mass points. In the other class, there is a gap in the price support where no price is chosen in the gap and there is an atom at the highest price chosen in equilibrium. In each equilibrium, there are two branches of the price distribution; if a seller chooses a price in the lower one, then he always makes a sale if the buyer's experiment is favorable to the seller and may make a sale even if the experiment is not favorable, depending on how high a price the other seller chosen in the upper branch. If a seller chooses a price in the upper branch, he only makes a sale if the experiment is favorable and if the other seller does not choose too low a price in the lower branch. Naturally, the equilibrium price distribution balances out a seller's incentives to make them indifferent to every price chosen with positive probability.

A key feature of the equilibrium is that the more precise the experiment conducted by a buyer, the higher the price and expected seller profit. The precision level chosen is based on what prices the buyer expects to face, while the prices are based on the sellers' *expectations* of a buyer's precision level. The more precise the experiment, the more sure is a buyer of which product is best, and this induces each seller to chooses higher prices because if the experiment turns out favorably, the buyer is willing to pay more. We find that the buyer will choose a more precise experiment the lower the cost of experimentation and the higher the level of product differentiation. These results lead to some interesting comparative statics. First, seller profit is always increasing in the level of differentiation and falling in a buyer's cost of experimenting. The comparative statics for the buyer welfare are more subtle. A higher level of product differentiation relaxes seller price competition, since the experiment is expected to be more precise, but at the same time increases the equilibrium spread of the distribution and thus increases a buyer's incentive to buy against signal. We find that the first effect dominates when differentiation is large, lowering buyer welfare, and the second when differentiation is small, raising buyer welfare. We also demonstrate that making it more costly for a buyer to experiment has an ambiguous effect on buyer welfare. On the one hand, there is the standard direct effect which lowers a buyer's payoff. On the other hand, there is an indirect effect which is that while lowering a buyer level of experimentation may lead to lower prices, it also leads to a lower probability of the buyer getting the right match.

In many markets, there is a dominant platform where a large fraction of transactions occurs on the platform; Amazon is a leading example.[1] We embed our basic model into a platform setting with a continuum of potential buyers with varying outside utility options. Before consumers choose their level of experiments and the firms choose their prices, the platform can choose its business model by either choosing what type of sellers to allow to sell on the platform, either ones that are relatively "standard" and do not add much from the base utility level or have products that are more "idiosyncratic" and more differentiated. The other choice that the platform can control is that it can reduce the cost for consumers to run an experiment, by making it easier for consumers to compare products. The platform makes revenue by collecting a percentage fee from each transaction carried out on it. Naturally, the platform cares both about the transaction price and the number of buyers that join the platform, where each instrument that the platform can control, the level of product differentiation or the cost of experimentation, effects both the level of consumer experimentation and the prices that firms charge.

We find that in many circumstances, the platform chooses its business model that displays the classic balance of a firm with market power that weighs the cost of raising price with the reduction in demand. In our setting, the platform does not directly control, but influences how consumers and firms will play the subsequent game and thus the number of consumers that will join the platform and the level of experimentation and prices that result. The subtle effects from our base model arise naturally in this setting. We also, show that the platform may choose to make experimentation very cheap, leading to very high firm prices.

Literature Our paper belongs to the growing literature studying the interaction of endogenous information disclosure and pricing initiated by Lewis and Sappington (1994), Anderson and Renault (2006) and Roesler and Szentes (2017), who study information design in monopoly settings to maximize either the seller's or the buyer's welfare. Armstrong and

[1] We note that this is not a model of Amazon which is a huge, complicated firm. We are capturing just one set of incentives that Amazon or other platforms face.

Zhou (2022) study information design in an oligopoly competition model with product differentiation. They derive ex ante optimal information structure for consumers and sellers. Elliott et al. (2021) also consider information design in an oligopoly setting and investigate how to send different information about the consumers to different sellers to affect competition. Also see Dogan and Hu (2022) who consider information design in canonical consumer search models. Ivanov (2013) and Hwang et al. (2019) consider a setting where sellers simultaneously choose prices and disclosure information about their products. That is, all these papers have a common feature where information design occurs first and then prices are subsequently determined. Some have firms that can commit to an information policy, while other have a third party choose the information design to maximize some objective function. On the contrary, our paper introduces consumer's endogenous learning into Moscarini and Ottaviani (2001)'s duopoly competition model and emphasizes the equilibrium interplay between price competition and endogenous product differentiation generated by consumers' costly information acquisition which occur simultaneously. That is, we do not rely on commitment by any agent in the game.

Ravid et al. (2022) consider a simultaneous-move game where the buyer chooses information acquisition and the seller chooses price. The focus is to argue an arbitrary small but positive information acquisition cost has a profound impact on equilibrium outcome. On the contrary, we study a duopoly model and focus on the interaction between firms' price competition and endogenous product differentiation formed by consumers' information acquisition choice. We also consider comparative statics with respect to information acquisition cost and consumers' sensitivity to product differentiation.

A paper subsequent to Ravid et al. (2022) by Albrecht and Whitmeyer (2023), contemporaneous with our paper, examines a duopoly model that shares some features of our paper. They also have firms choosing prices simultaneously at the same time consumers choose information. Their focus is on the comparison with Ravid et al. (2022), where they find as the cost of acquiring information goes to 0, that this is equivalent to the case where the consumer has the information for free. They, like us, find that consumer utility is non-monotonic in the cost of acquiring information. They differ from us in how consumer preferences are modeled and do not focus on product differentiation as we do.

A well-known alternative to model information friction is to introduce consumer search, where there is a prominent industrial organization literature studying price competition in the presence of both homogeneous products (Burdett and Judd, 1983; Stahl, 1989) and product differentiation (Wolinsky, 1986; Anderson and Renault, 1999) and consumer search.

These models also have simultaneous actions by consumers and firms as we do. One of the key differences is that that the perceived level of product differentiation in the search papers is exogenous, while in our setup it is endogenous and chosen by the consumer. Furthermore, in our setting the consumer can learn the price quotes of each firm for free, while there is a cost for each new price quote in the search models.[2]

Organization The rest of the paper is organized as follows. Section 2 sets up the model, and Section 3 analyzes the model. Specifically, in Section 3.1, we derive the buyer's demand function given her information acquisition and sellers' prices. In Section 3.2, we take the demand function as given and fully characterize the unique symmetric equilibrium in sellers' pricing game. In section 3.3, we take sellers' symmetric pricing rule as given and derive the buyer's optimal information acquisition policy. Section 3.4 combines sellers' and buyer's best responses and show that there exists a unique symmetric equilibrium. Section 3.5 studies the comparative statics with respect to the change in key parameters and the welfare implications. Section 4 applies our theory to understand the trade-off of platform. Section 5 concludes. Omitted proofs and additional results are relegated to Appendices.

2 Model

Players We consider a spatial (horizontal) competition model with *binary-type* consumers. There are three players: a representative buyer and two sellers, $i = 1, 2$. Sellers produce differentiated products at zero cost and post prices p_1 and p_2, respectively. Seller i's payoff is p_i if the buyer purchases his product; otherwise, it is 0. The buyer has unit demand. If she purchases from seller i at price p_i, she gets a payoff

$$v + t\mathbb{I}(\theta = i) - p_i, \tag{1}$$

where $v > 0$ reflects the buyer's base utility of consuming either good 1 or 2. The parameter $\theta \in \{1, 2\}$ represents the buyer's *taste* where the indicator function $\mathbb{I}(\cdot)$ captures whether the product is a good match for the buyer. It takes on a value of one if the buyer's taste matches the product, i.e., $\theta = i$ and zero if the buyer's taste does not. The parameter $t > 0$ reflects

[2]In the literature on order search, consumers' search is directed by prices which they observe for free, and sellers have the incentive to set price to attract consumers' visit. See, e.g., Armstrong (2017) and Choi et al. (2018).

the buyer's sensitivity to product differentiation. If the buyer does not purchase from any seller, she receives zero payoff in the base model.

Information Acquisition The buyer's taste is initially unknown and all players believe $\Pr(\theta = 1) = 0.5$. However, the buyer can incur a cost to acquire information about the value of θ. For simplicity, we assume that the buyer can choose $\gamma \in [0, 1]$ and observe a *signal* $s = \{1, 2\}$ according to a *symmetric experiment* such that

$$\Pr(s|\theta) = \begin{cases} \frac{1+\gamma}{2} & \text{if } s = \theta \\ \frac{1-\gamma}{2} & \text{if } s \neq \theta \end{cases},$$

where γ captures the precision of the experiment. It is straightforward to see that the experiment becomes more informative in the sense of Blackwell (1953) as γ increases. The informativeness of the experiment determines the buyer's *perceived product differentiation* between two sellers. In an extreme case where $\gamma = 0$, the buyer views the two products as perfect substitutes. In the other extreme case where $\gamma = 1$, the perceived production differentiation reflects the objective payoff difference between the two products. When $\gamma \in (0, 1)$, the buyer partially and imperfectly perceives the product differentiation between the two sellers.

The buyer's cost of information acquisition is a function of the precision, $\kappa c(\gamma)$ where the parameter $\kappa \geq 0$ captures how costly it is for the buyer to reduce uncertainty about her own taste. Naturally, we assume that $c(\cdot)$ is twice-differentiable, strictly increasing, strictly convex with $c(\gamma) = 0$ if and only if $\gamma = 0$. We also impose the following conventional conditions to ensure interior solutions, $c'(0) = 0, c'(1) = +\infty$. The meaning of these properties is self-evident, and they are satisfied by many popular specifications used in the literature. One popular example, following the seminal work of Sims (1998) on rational inattention, is to assume that $c(\gamma)$ equals the reduction of Shannon's entropy (or the expected Kullback-Leibler difference between the buyer's prior and posterior belief). In a recent paper, Pomatto et al. (2020) employ an axiomatic approach and propose the following log-likelihood ratio (LLR) cost function

$$c(\gamma) = \frac{1+\gamma}{2} \ln \frac{1+\gamma}{1-\gamma} + \frac{1-\gamma}{2} \ln \frac{1-\gamma}{1+\gamma}, \tag{2}$$

which is the expected log-likelihood ratio between state θ and θ' when the true state is θ. The cost function captures a number of natural properties including (i) experiments that are more informative in the sense of Blackwell (1953) cost more and it has been used in many applied works such as the moral hazard principal agent model, see Milgrom (1981); (ii) the cost of generating two independent signals is the sum of their costs, and generating a signal with probability half costs half its original. [3] We will use this cost function to conduct some numerical simulations.

Extensive Form, Strategies, and Equilibrium The timing of the game is as follows.

- At the first stage, the buyer and both sellers move simultaneously. The buyer chooses the precision of her experiment, and sellers choose their prices, respectively.
- At the second stage, nature randomly assigns the taste of the buyer and generates a signal realization according to the experiment chosen by the buyer at stage one. The buyer observes the sellers' prices and a signal from her chosen experiment, and decides whether and where to purchase.

We allow sellers to play mixed strategies, and denote by F_i as seller i's strategy. The buyer's strategy specifies the precision of information acquisition γ and if she purchases a good which seller given her received signal and prices, i.e., $\sigma(s, p_1, p_2) \in \Delta(\{0, 1, 2\})$ where 0 represents no purchase and $i = 1, 2$ represents purchasing from seller i.

We focus on *symmetric* solution concepts where $F_1 = F_2$. Formally, a *symmetric perfect Bayesian equilibrium* is a triple (γ, σ, F) where (i) σ maximizes the buyer payoff given γ, p_1, p_2, (ii) γ maximizes the buyer's expected payoff given F, σ, and (iii) F maximizes each seller's expected payoff given the buyer's strategy being γ, σ and his rival seller's strategy being F.

Discussion We assume that the buyer acquires information at the same stage when sellers post prices. This is a departure from much of the information design literature in general and in particular with information gathering by consumers, where there is a strong level of commitment by one of the parties. In the case where buyers acquire information, it is a commitment on the price by the seller. While there are settings, where strong the ability to commitment seems reasonable, this is not always going to be the case and the seller would like to change their price after the buyer has acquired information. We view our assumption is reasonable when obtaining bids from contractors for renovations and for buying many

[3] The functional form is also supported by the implications of Morris and Strack (2019). They study a continuous-time Wald problem where a decision maker sequentially observes signals at a constant flow cost and decides when to stop. They find the expected stopping time (total cost) is consistent with the form in (2).

durable goods. Consumers make decisions on a wide range of products that are often quite independent from each other. They buy bicycles, HVAC systems, computer monitors, etc. To help inform their decisions, consumers often subscribe to services. Some are general "all purpose" services such as Consumer Reports and the New York Times for its Wirecutter feature. These are useful and relatively low cost per use for many goods that a consumer will want to purchase. They give a consumer a good overview of how to compare products and can be thought of as low cost, relatively not too precise experiments. Other information services are quite specialized. Subscriptions to Car and Driver, Wired, and Bicycling Magazine are much more specialized and provide much more information for purchases. These can be thought of as relatively high cost, quite precise experiments. We think of our consumers as choosing their levels of informativeness, precision of experiments, as a combination of all these services. Furthermore, we assume that after the consumer observes the prices offered by the sellers and the outcome of the experiment, then the cost of future experiments for any particular are too high relative to the potential value of the experiment.[4] Thus, we assume that the information gathering process takes a relatively long time for the consumer to digest for the complicated, infrequently purchased goods that are the focus of this study. Once the consumer observes the firms' prices, it is too costly to gather additional information.

A second way to think about it is in many markets, a buyer acquires information before knowing the terms of trade. For example, in many over-the-counter markets, it is costly to get a price quote, and a consumers often rely on online resources to first learn what types of products better fit them before going through the process of getting price quotes. Finally, one can think of the equilibrium outcome as a long-term steady state where new consumers enter every period and acquire information and sellers rationally set prices according to their equilibrium strategies given the correct expectation of consumers' information acquisition.

3 Analysis

We use backward induction to analyze the problem. First, we derive the buyer's optimal purchase decision and sellers' demand function given γ. Second, given the demand function and γ, we derive sellers' best response. Third, given sellers' pricing strategies, we investigate the buyer's optimal information acquisition. Finally, we piece together the problem of the buyer and the sellers to characterize the unique (symmetric) equilibrium.

[4] This is similar in spirit to the assumption often made in search models that the first search is free in order to get around the Diamond Paradox.

3.1 Buyer's Purchase Rule and Demand

We begin with the buyer's purchase rule and derive sellers' demand functions. Upon receiving the signal, the buyer updates her belief about θ accordingly. Given a signal s, if the buyer purchases from seller i, her expected payoff is

$$\begin{cases} v + t\frac{1+\gamma}{2} - p_i & \text{if } i = s \\ v + t\frac{1-\gamma}{2} - p_i & \text{if } i \neq s \end{cases}, \tag{3}$$

and she finds it optimal to purchase from some seller if and only if

$$v + \max\left\{ t\frac{1+\gamma}{2} - p_s, t\frac{1-\gamma}{2} - p_{-s} \right\} \geq 0, \tag{4}$$

where p_s denotes the price charged by seller $i = s$, whereas p_{-s} denotes the price charged by seller $i \neq s$. In other words, seller i's demand is

$$D_i(p_i, p_{-i}, \gamma) = 0,$$

if inequality (4) fails for $i = 1, 2$. Hence, it is without loss to assume that sellers will only choose price from the compact interval $[0, v + t(1+\gamma)/2]$ for any γ.

Suppose that inequality (4) holds and consider an arbitrary $\gamma \geq 0$. Since the signal is weakly informative, this will affect the buyer's purchase decision. Upon receiving signal s, the buyer purchases *following the signal (or obediently)*, i.e., from seller $i = s$, if

$$v + t\frac{1+\gamma}{2} - p_s \geq v + t\frac{1-\gamma}{2} - p_{-s},$$

and purchases *against the signal (or disobediently)*, i.e., from seller $i \neq s$, otherwise. As a result, given γ and his rival's price p_{-i}, seller i's demand function is

$$D_i(p_i | p_{-i}, \gamma) = \begin{cases} 1 & \text{if } p_i - p_{-i} < -t\gamma \\ 0.5 & \text{if } p_i - p_{-i} \in [-t\gamma, t\gamma] \\ 0 & \text{if } p_i - p_{-i} > t\gamma \end{cases}, \tag{5}$$

for $(p_i, p_{-i}) \in [0, v + t(1+\gamma)/2]^2$. Notice that when the difference between two prices is less than $t\gamma$, the buyer purchases obediently, and so sellers equally split the market. When

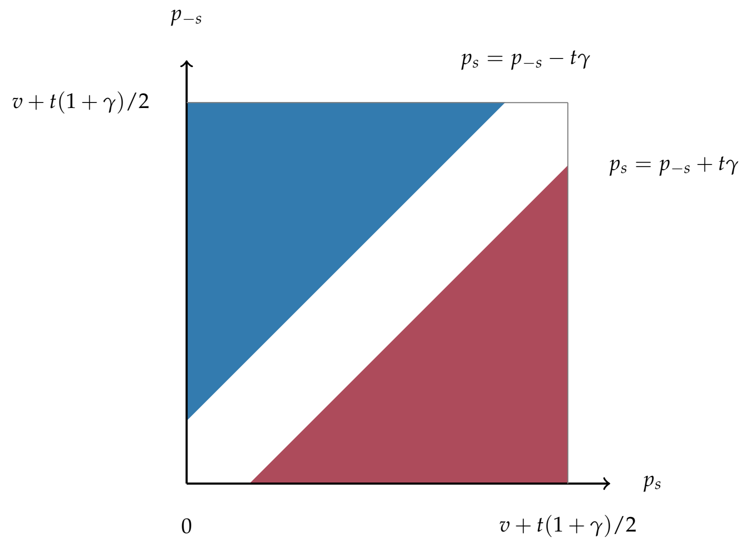


Figure 1: Demand for two sellers when $\gamma > 0$. In the upper triangle blue area, $D_i = 1, D_{-i} = 0$, in the lower triangle red area, $D_i = 0, D_{-i} = 1$, and in between $D_i = 0.5, D_{-i} = 0.5$. When $\gamma \to 0$, the middle area vanishes and we converge to Bertrand demand.

$\gamma \to 0$, the signal becomes uninformative, the buyer views the two products as almost perfect substitutes, and so two sellers split the market equally only if they charge the same price. In sum, seller i's demand function is given by equation (5), which is visualized in Figure 1. [5]

3.2 Sellers' Pricing Game

Given the demand function we specified in (5) and an arbitrary $\gamma \in [0, 1]$, we can define a *pricing game* between two sellers who simultaneously choose their prices. This model is similar to Moscarini and Ottaviani (2001) who allow the consumer's prior belief to be different from 1/2. A (mixed-strategy) Nash equilibrium of the game is a pair $\{F_i(p)\}_{i=1,2}$ such that

$$p \in \arg\max \int pD_i(p, \tilde{p}, \gamma)dF_{-i}(\tilde{p}),$$

for each $p \in Supp(F_i)$ and $i = 1, 2$. The equilibrium is *symmetric* if $F_1(\cdot) = F_2(\cdot)$.

In what follows, we fully characterize the unique equilibrium of the sellers' pricing

[5] The demand function is discontinuous and sellers compete *a la* Hotelling (1929). As we shall show, the equilibrium price distribution shares some similarity to Osborne and Pitchik (1987) and Moscarini and Ottaviani (2001) where the sellers play mixed strategies in the equilibrium.

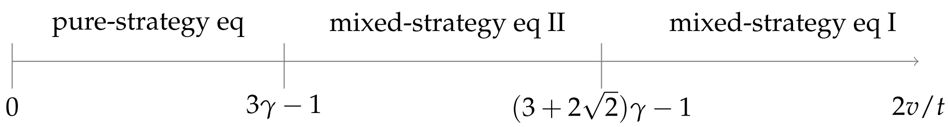


Figure 2: Equilibrium in Pricing Game.

game.[6] When $\gamma = 0$, there is a unique pure strategy equilibrium, which is Bertrand and prices are equal to 0. When $\gamma > 0$, the equilibrium may take three possible forms, depending on the value of v/t and γ: either a pure strategy equilibrium or one of two different mixed strategy equilibrium structures. This is illustrated in Figure 2. If γ is larger than some critical level, then for each γ above this critical cutoff, there exists a set of preferences with sufficiently large product differentiation, v/t sufficiently small (less than $3\gamma - t < 2v/t$), then the unique equilibrium is in pure strategies. On the other hand, if γ is below the critical cutoff, then there does not exist a pure strategy equilibrium for any v/t. For $2v/t > (3+2\sqrt{2})\gamma - 1$, there is a unique equilibrium which is in mixed strategy and has no mass point. While when $3\gamma - t < 2v/t \leq (3+2\sqrt{2})\gamma - 1$, there is a unique equilibrium which is in mixed strategies and has a mass point only at the upper bound of the support. We first prove the conditions for a pure strategy equilibrium.

Lemma 1. *The pricing game has a unique Nash equilibrium which is in pure strategies if and only if one of the following conditions holds: (i) either $\gamma = 0$, or (ii) $\gamma > 0$ and*

$$t(3\gamma - 1) \geq 2v. \tag{6}$$

When $\gamma = 0$, the equilibrium strategy is to choose $p_1 = p_2 = 0$. When $\gamma > 0$ and condition (6) holds, the equilibrium strategy is to choose $p_1 = p_2 = v + t(1+\gamma)/2$.

In the first case where $\gamma = 0$, the buyer fails to perceives the product differentiation between sellers, and so sellers compete in the sense of Bertrand. As a result, the only Nash equilibrium is to price at marginal cost.

In the second case where $\gamma > 0$, the buyer perceives product differentiation between two sellers. Upon receiving signal $s \in \{1, 2\}$, the buyer leans toward purchasing obediently if seller s's price is not too high relative to his rival. Condition (6) says that the product differentiation between two sellers is sufficiently substantial relative to the base utility v and

[6] Notice that the equilibrium existence is not guaranteed by standard argument because sellers' payoffs are discontinuous (Dasgupta and Maskin, 1986).

the buyer's signal is sufficiently informative for her to perceive such a difference. In this case, sellers are endowed with a substantial amount of market power that softens competition. As a result, each seller will focus on the buyer receiving a signal that favors him, always acting obediently, and extracts the full surplus. That is, the seller can act like a monopolist and the buyer is held to her reservation utility. The only possible deviation is that a seller may lower the price by slightly more than $t\gamma$ to steal the business from his rival. However, this is not profitable, i.e.,

$$\frac{1}{2}\big[v + t\frac{1+\gamma}{2}\big] \geq v + t\frac{1-\gamma}{2},$$

due to condition (6). Thus, it must be the case that there is enough product differentiation such that a seller is not tempted to lower his price enough to overcome an unfavorable signal.

To see that there exists no pure strategy equilibrium when neither of the above conditions holds, we rely on a proof by contradiction. Suppose there exists a pure-strategy equilibrium (p_1, p_2). It is without loss to assume that $p_i \leq v + t\frac{1+\gamma}{2}$ for $i = 1, 2$ to satisfy the buyer's participation constraint (4). If $p_i - p_{-i} > \gamma$ for $i = 1, 2$, then seller i's demand is zero, and so he has an incentive to deviate. If $-\gamma \leq p_i - p_{-i} \leq \gamma$ for some $i = 1, 2$, the buyer purchases obediently, then two sellers equally split the market. In this case, seller $-i$ has the incentive to slightly increase his price such that $p_{-i} - p_i < \gamma$ holds. By doing so, his demand is still $1/2$ as long as the buyer's participation constraint (4) holds. Using this logic, we can conclude that whenever $\gamma > 0$, the only possible pure-strategy equilibrium is $p_1 = p_2 = v + t(1+\gamma)/2$. However, such a price profile fails to be a Nash equilibrium when condition (6) is not satisfied.

Lemma 1 discusses two extreme cases. In the first case, the buyer does not perceive any product differentiation and sellers extract no surplus; whereas in the second case, sellers leave no expected surplus to the buyer. We devote the rest of this section to the scenario where $\gamma > 0$ and condition (6) fails. As in Moscarini and Ottaviani (2001), there exists a unique equilibrium which is mixed-strategy and sellers extract some of the surplus from the buyer.

Lemma 2. *Suppose that* $\gamma > 0$ *and*

$$2v \geq t\left[(3 + 2\sqrt{2})\gamma - 1\right]. \tag{7}$$

The pricing game has a unique symmetric Nash equilibrium which is in mixed strategies. In equilib-

rium, sellers randomize according to an atomless CDF,

$$F(p) = \begin{cases} 1 - \frac{2\pi}{p+t\gamma} & \text{if } p \in [\underline{p}, \bar{p} - t\gamma) \\ 2 - \frac{2\pi}{p-t\gamma} & \text{if } p \in [\bar{p} - t\gamma, \bar{p}] \end{cases}, \tag{8}$$

where $\bar{p} = t\gamma(2 + \sqrt{2})$, $\underline{p} = \bar{p} - 2t\gamma = t\gamma\sqrt{2}$ *and*

$$\pi = \frac{t\gamma(1 + \sqrt{2})}{2},$$

is each seller's equilibrium profit.

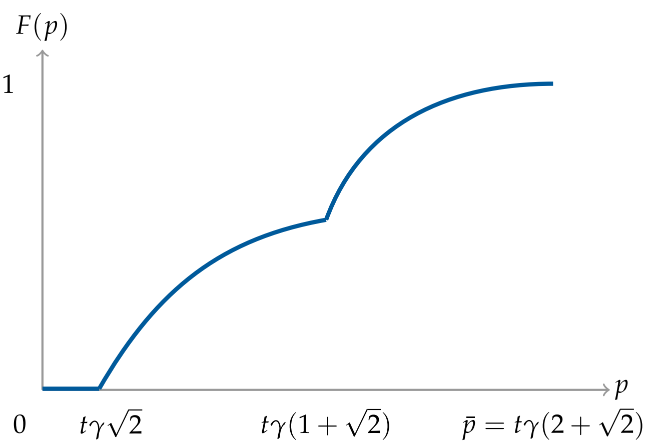


Figure 3: Mixed-Strategy Equilibrium I CDF.

Condition (7) ensures that any $p \in [\underline{p}, \bar{p}]$ satisfies the buyer's participation constraint (4). It holds when v/t is sufficiently large. This is similar to the full-market coverage assumption made in the literature: the product differentiation between two sellers exists but is not substantial enough relative to the base utility v. The argument of the equilibrium existence and unique follows Moscarini and Ottaviani (2001). In what follows, we give a heuristic argument why expression (8) is an equilibrium to deliver some intuition.

To sustain a mixed-strategy equilibrium, we need to guarantee that (i) each seller must be indifferent to charge any price on the interval $[\underline{p}, \bar{p}]$, and (ii) no seller has the incentive to deviate to $p \notin [\underline{p}, \bar{p}]$. We begin with sellers' indifference condition. The interval $[\underline{p}, \bar{p}]$ can be equally divided into two parts: the high price region where $\bar{p} - t\gamma \leq p \leq \bar{p}$ and the low price region where $\underline{p} \leq p < \bar{p} - t\gamma$, which are visualized in Figure 4. As it is standard in the literature of sales, e.g., Varian (1980) and Burdett and Judd (1983), such an indifference comes from the trade off between price and demand. In our case, the seller's indifference condition in the high (or low) price region disciplines the equilibrium strategy distribution in the low (or high) price region. Due to the presence of the buyer's perceived product differentiation, a marginal change in his price p will affect a seller's demand if and only if his rival's price is in the neighborhood of $p - t\gamma$ or $p + t\gamma$.[7]

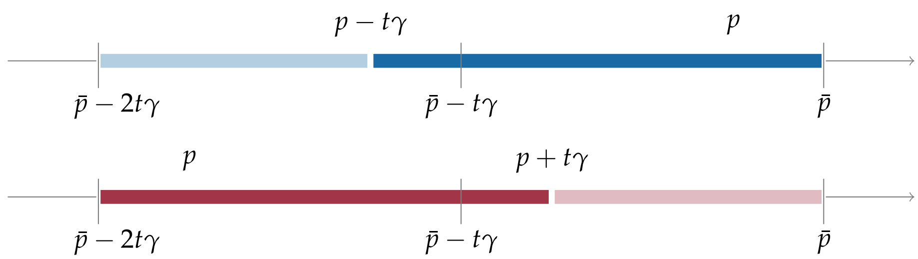


Figure 4: In the upper panel, a seller chooses a price $p \in [\bar{p} - t\gamma, \bar{p}]$. His demand is 0.5 (or 0) if his rival's price is above (or below) $p - t\gamma$. In the lower panel, a seller chooses a price $p \in [\bar{p} - 2t\gamma, \bar{p} - t\gamma]$. His demand is 0.5 (or 1) if his rival's price is below (or above) $p + t\gamma$.

If a seller chooses a price in the high price region, his payoff is

$$\pi = p\Big[\int_{p-t\gamma}^{\bar{p}} \frac{1}{2} dF(\tilde{p}) + \int_{\bar{p}-2t\gamma}^{p-t\gamma} 0 dF(\tilde{p})\Big] = \frac{p}{2}\Big[1 - F(p - t\gamma)\Big].$$

That is, the seller can never get the entire market, but he can have half of the market as long as his rival's price is greater than $p - t\gamma$ (see Figure 4). Replacing $\tilde{p} = p - t\gamma$ immediately generates the expression of $F(\cdot)$ for any $\tilde{p} \in [\bar{p} - 2t\gamma, \bar{p} - t\gamma]$. Similarly, if a seller charges a price in the low-price region, his payoff is

$$\pi = p\Big[\int_{\bar{p}-2t\gamma}^{p+t\gamma} \frac{1}{2} dF(\tilde{p}) + \int_{p+t\gamma}^{\bar{p}} 1 dF(\tilde{p})\Big] = p\Big[1 - \frac{1}{2}F(p + t\gamma)\Big].$$

In this case, the seller always secures the demand from half of the market, the buyer gets a favorable signal, and he may even get all the sales if the buyer gets an unfavorable signal and his rival's price is greater than $p + t\gamma$. Simple algebra gives the expression in (8) for $\tilde{p} \in [\bar{p} - 2t\gamma, \bar{p} - t\gamma]$. Since the expression in $F(\cdot)$ is atomless, the continuity argument and the standard property of CDF help to solve for $\bar{p}$ and π.

[7] This feature makes our analysis more aligned to the mixed-strategy equilibrium appearing in the traditional Hotelling setting (Osborne and Pitchik 1987) than Varian (1980).

Next, we show that no seller has the incentive to deviate outside the equilibrium strategy. If a seller unilaterally deviates to a price *above* the equilibrium price interval, i.e., $p > \bar{p}$, his payoff is

$$\pi^a(p) = \begin{cases} 0 & \text{if } p \geq \bar{p} + t\gamma \\ \frac{1}{2}p[1 - F(p - t\gamma)] & \text{if } p \in (\bar{p}, \bar{p} + t\gamma) \end{cases}.$$

which are both less than the equilibrium profit. On the other hand, if a seller unilaterally deviates to a price *below* the equilibrium price interval, i.e., $p < \bar{p} - 2t\gamma$, his payoff is

$$\pi^b(p) = \begin{cases} p & \text{if } p \leq \bar{p} - 3t\gamma \\ \frac{p}{2}[2 - F(p + t\gamma)] & \text{if } p \in (\bar{p} - 3t\gamma, \bar{p} - 2t\gamma) \end{cases}$$

We can plug the expression of $F(\cdot)$ into the deviation payoff and show that $\pi^a(\cdot)$ decreases in p and achieves its maximum at $\bar{p}$, and $\pi^b(p)$ increases and achieves its maximum at $\bar{p} - 2t\gamma$. The intuition is standard as in most models of sales, e.g., Varian (1980). The equilibrium distribution $F(\cdot)$ is constructed to let the change of demand and the change of price exactly offset on $[\bar{p} - 2t\gamma, \bar{p}]$. When a seller deviates either above or below the equilibrium support, the balance between two effects is broken, and so he either ends up losing too much demand or cutting price too much, resulting in a strictly lower profit.

In sum, it is not profitable to deviate to a price either above or below the equilibrium price interval. So, we conclude that when $\gamma > 0$, and conditions (6) and (7) hold, the strategy described in Lemma 2 is an equilibrium of the pricing game.

The following results are straightforward.

Corollary 1. *As γ increases, the equilibrium price's expectation and variance increase, and sellers' equilibrium profit increases.*

In the pricing game, a buyer's extra willingness to pay to purchase following the signal is increasing in γ. As a result, as γ increases, sellers enjoy more market power and charge a higher price in expectation. The seller's equilibrium profit is obviously increasing from its expression in Lemma 2. In Appendix B, we derive the formula of the expected equilibrium price and show it is increasing in γ.

The reader will notice that there is a gap of parameters that we have not covered yet for the seller pricing game. We cover this gap in Appendix A and show that the equilibrium pricing distributions will also have two branches as in the large v/t case, where a price in the lower branch will always have the buyer making a purchase with a favorable signal and may have the buyer making a purchase even if there is an unfavorable signal, while a price in the upper branch will only be selected if the signal is favorable and the rival's price is not too low. The difference for this parameter configuration is that a gap separates the two branches and a mass point at the highest equilibrium price. For ease of exposition and because for our comparative statics we focus on large v/t cases, we relegate the analysis of small v/t case to Appendix A. We also show that, as v and t change, the equilibrium profit and price (range) respond monotonically and "continuously" in this regime.

3.3 Buyer's Information Acquisition

Given the buyer's purchase strategy σ and the sellers' symmetric pricing strategy, we derive the buyer's optimal information acquisition choice.

Lemma 3. *Suppose that sellers play symmetric pure strategies $p_1 = p_2 = p$.*

1. *If $p \leq p^* \equiv v + t\frac{1+\gamma^*}{2} - \kappa c(\gamma^*)$ where γ^* solves*

$$\max_{\gamma \in [0,1]} t\frac{\gamma}{2} - \kappa c(\gamma), \tag{9}$$

   the buyer's optimal precision is γ^*.

2. *If $p > p^*$, the buyer's optimal precision is 0.*

The value function of programming (9) represents the maximum value of information acquisition, and its lower bound is zero. If the price is lower than p^*, the buyer chooses precision γ^*, purchases following the signal, and gets a payoff $p^* - p \geq 0$, which automatically satisfies her participation constraint (4). If $p > p^*$, the buyer acquires no information since even the maximum value of information acquisition is insufficient to compensate for the high price, and so he perceives no difference between two sellers' products. At the second stage, if both sellers price above $v + t/2$, the buyer won't purchase; otherwise, he purchases from the seller with lower price (a tie is broken randomly).

Lemma 3 tells us if there is a symmetric equilibrium, sellers must play mixed strategies in equilibrium. This is due to Lemma 1. For now, we focus on the case where v/t is sufficiently large so that for any $\gamma \in [0, 1]$, condition (2) holds, and the buyer's participation constraint holds for any $p \in supp(F)$. Formally,

Assumption 1.

$$v/t > 1 + \sqrt{2}, \tag{10}$$

Suppose that each seller randomizes according to an atomless CDF,

$$F^{\hat{\gamma}}(p) = \begin{cases} 1 - \frac{t\hat{\gamma}(1+\sqrt{2})}{p+t\hat{\gamma}} & \text{if } p \in [t\hat{\gamma}\sqrt{2}, t\hat{\gamma}(1+\sqrt{2})) \\ 2 - \frac{t\hat{\gamma}(1+\sqrt{2})}{p-t\hat{\gamma}} & \text{if } p \in [t\hat{\gamma}(1+\sqrt{2}), t\hat{\gamma}(2+\sqrt{2})] \end{cases}, \tag{11}$$

for some $\hat{\gamma} \in [0,1]$. The buyer's best response in information acquisition must solve the following problem

$$\max_{\gamma \in [0,1]} v + b(\gamma, \hat{\gamma}) - \kappa c(\gamma), \tag{12}$$

where $b(\gamma, \hat{\gamma})$ is the buyer's value of information acquisition with precision γ given sellers follow strategy $F^{\hat{\gamma}}$, i.e.,

$$b(\gamma, \hat{\gamma}) = \int_{P_{\gamma,\hat{\gamma}}} \left[t\frac{1+\gamma}{2} - p\right] dF^{\hat{\gamma}}(p') dF^{\hat{\gamma}}(p) + \int_{Q_{\gamma,\hat{\gamma}}} \left[t\frac{1-\gamma}{2} - p'\right] dF^{\hat{\gamma}}(p) dF^{\hat{\gamma}}(p'), \tag{13}$$

where p denotes the realized price of the seller who is recommended by the signal, while p' is the price of the seller that is not recommended by the signal. $P_{\gamma,\hat{\gamma}}$ and $Q_{\gamma,\hat{\gamma}}$ are the sets of price pairs such that the buyer purchases obediently and disobediently, respectively.

$$\begin{aligned} P_{\gamma,\hat{\gamma}} &= \left\{(p,p') \in \left[t\hat{\gamma}\sqrt{2}, t\hat{\gamma}(2+\sqrt{2})\right]^2 : p \le p' + t\gamma\right\}, \\ Q_{\gamma,\hat{\gamma}} &= \left\{(p,p') \in \left[t\hat{\gamma}\sqrt{2}, t\hat{\gamma}(2-\sqrt{2})\right]^2 : p' \le p - t\gamma\right\}. \end{aligned}$$

The intuition behind expressions in (12) and (13) is as follows. Since we assume full market coverage, the buyer always purchases as long as the prices are drawn from $F^{\hat{\gamma}}$. Hence, by choosing precision γ, the buyer's payoff includes the base utility from the purchase v, the disutility due to information acquisition $\kappa c(\gamma)$, and value of information $b(\gamma, \hat{\gamma})$.

It is necessary to further elaborate the expression of $b(\gamma, \hat{\gamma})$. Due to the symmetric specification of information acquisition and the assumption of uniform prior, the buyer will receive a signal that favors each seller with equal probability. The buyer will purchase following the signal and receive a payoff $t\frac{1+\gamma}{2} - p$; otherwise, if $(p, p') \in Q_{\gamma,\hat{\gamma}}$, the buyer will purchase against the signal and receive a payoff $t\frac{1-\gamma}{2} - p'$. The idea is visualized by Figure 5. It is immediate that $b(\cdot, \hat{\gamma})$ is increasing due to Blackwell (1953). Moreover,

Lemma 4. *If the seller's believed experiment precision of the buyer is correct, i.e., $\gamma = \hat{\gamma}$, the marginal*

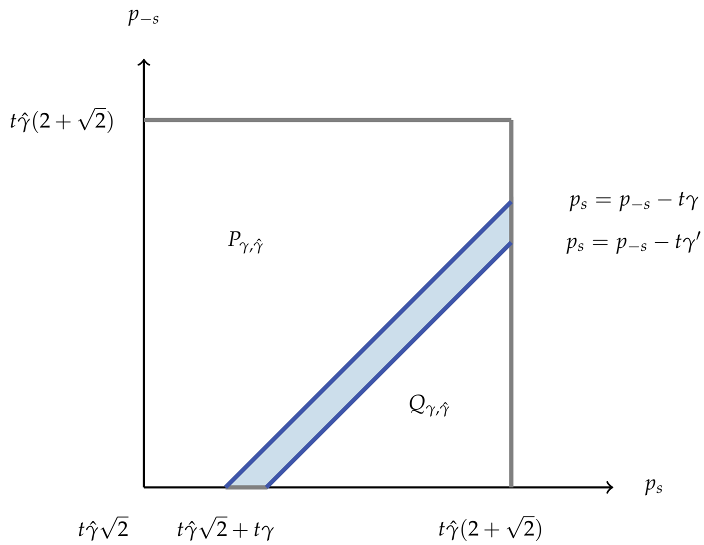


Figure 5: Buyer's Purchase Rule. We denote p_s as the price of the seller favored by the buyer's signal and p_{-s} as the price of the other seller. Since $\hat{\gamma}$ is fixed, the support of (p, p') is fixed. Also because we assume v/t to be sufficiently large, increasing γ to γ' shifts down the buyer's price indifferent curve $p_s = p_{-s} - t\gamma$ to $p_s = p_{-s} - t\gamma'$, and the regions of $P_{\gamma,\hat{\gamma}}, Q_{\gamma,\hat{\gamma}}$. When $\gamma = 0$, the price indifferent curve coincides with the 45-degree line. When $\gamma > 2\hat{\gamma}$, the information is so precise that the buyer never purchases against the signal, i.e., $Q_{\gamma,\hat{\gamma}} = \emptyset$.

effect of increasing experiment precision is positive and linear in t; i.e.,

$$b_1(\gamma, \gamma) = \beta_1 t > 0 \tag{14}$$

for some $\beta_1 > 0$, and the marginal effect of increasing seller's believed experiment precision can be either positive or negative and is affine in t; i.e.,

$$b_2(\gamma, \gamma) = \beta_0 - \beta_2 t \tag{15}$$

where $\beta_0, \beta_2 > 0$.

The algebra behind Lemma 4 is rather complicated, so we leave it to Appendix B. But the expression of (14) is intuitive. Increasing the experiment precision induces the buyer to choose the correct good more frequently when everything else is constant, and making correct choices is more valuable when the buyer is more sensitive to product differentiation; i.e., when t is larger. Expression (15) implies that the buyer's payoff increases in the sellers' believed experiment precision if $t < \beta_0/\beta_2$ but decreases otherwise. To see the intuition, note that increasing sellers' belief about the buyer's experiment precision has two effects. On the one hand, by Corollary 1, increasing $\hat{\gamma}$ raises the equilibrium price on average, making the buyer worse off. On the other hand, it will also increase the spread of the equilibrium price dispersion, and the buyer may buy against the signal more frequently for a lower price. When t is big, the buyer has strong incentive to buy following the signal, and so the first effect dominates. When t is small, the buyer has less incentive to buy following the signal, and the second effect dominates. The proof is relegated to Appendix B.

3.4 Equilibrium Characterization

Now we are ready to provide the equilibrium characterization.

Proposition 1. *There is no equilibrium where sellers play pure strategies.*

Proof. By Lemma 1, an equilibrium where sellers play pure strategies exists under two scenarios. We will rule out these two possibilities. First, suppose that there is an equilibrium where $\gamma^* > 0$ and sellers extract all the surplus from the trade. The buyer has a profitable deviation by not acquiring information and not purchasing, which is a contradiction. Second, suppose that there is an equilibrium where $\gamma^* = 0$ and sellers compete a la Bertrand. However, the buyer can strictly benefit by choosing a positive but small γ because $c'(0) = 0$, a contradiction! □

The proposition implies that no pure-strategy equilibrium can be sustained for finite κ. Later we will show that in the limit case where $\kappa \to \infty$, the mixed-strategy equilibrium price will converge to the Bertrand equilibrium, $p = 0$.

Notice that, by Lemma 1, when a pure-strategy equilibrium of the seller's pricing game exists, sellers extract either no surplus from the buyer or the full surplus. The non-existence of pure-strategy equilibrium in Proposition 1 says that, under a mild condition, sellers and the buyer split the value of information acquisition in any symmetric equilibrium.

Therefore, if an equilibrium exists, sellers must play mixed strategies. The rest of the subsection is devoted to showing such an equilibrium exists and is unique under certain conditions. To focus on the most interesting economic interplay, we follow the tradition of the literature and assume full market coverage: v/t is sufficiently large so that the buyer always purchases from a seller in the equilibrium regardless of her choice of experiment precision, i.e., condition (10) holds. In such an equilibrium, sellers believes the buyer's choice of precision is $\hat{\gamma}$, and therefore play the equilibrium strategy of the pricing game specified by equation (11). If an equilibrium exists, the equilibrium precision γ^* must solve

$$\gamma^* \in G(\gamma^*), \tag{16}$$

where G is the solution correspondence of program (12). We are unable to prove that G is single-valued or convex-valued for arbitrary κ. Fortunately, the equilibrium existence and uniqueness can be proved.

Proposition 2. *Suppose that v/t is sufficiently large, i.e., inequality (10) holds. There exists a unique (symmetric) equilibrium where the equilibrium precision $\gamma^* \in (0,1)$ satisfies*

$$b_1(\gamma,\gamma) \equiv \beta_1 t = \kappa c'(\gamma), \tag{17}$$

where β_1 is specified in Lemma 4, and sellers randomize according to expression (11) with $\hat{\gamma} = \gamma^*$

Proof. First, if an equilibrium exists, the equilibrium precision $\gamma^* \in (0,1)$. To see this, note that we assume $c'(0) = 0$ and $b(\cdot,0)$ is strictly increasing, so $\gamma^* \neq 0$. Also, by assumption $c'(1) = \infty$, so $\gamma^* \neq 1$. If an interior equilibrium $\gamma^* \in (0,1)$ exists, it is determined by the solution to (17) by combining the buyer's first-order condition and the equilibrium condition. The existence of an interior solution is ensured by the intermediate value theorem

$$\begin{aligned} b_1(0,0) - \kappa c'(0) &> 0 \\ b_1(1,1) - \kappa c'(1) &< 0 \end{aligned}$$

under the assumptions that $c'(0) = 0$, $c'(1) = \infty$, and the continuity of $b_1(\cdot)$ and $c'(\cdot)$.

To see the uniqueness, notice that

$$\kappa c''(\gamma) - \frac{\partial b_1(\gamma,\gamma)}{\partial \gamma} = \kappa c''(\gamma) > 0$$

where $\frac{\partial b_1(\gamma,\gamma)}{\partial\gamma} = 0$ because $b_1(\gamma,\gamma)$ is a constant for any γ by Lemma 4. This implies that function $b_1(\gamma,\gamma) - \kappa c'(\gamma) = 0$ has exactly one solution in $\gamma \in [0,1]$. □

3.5 Comparative Statics and Welfare Analysis

This section first studies the comparative statics with respect to the buyer's sensitivity to product differentiation t and cost of information acquisition κ, and the corresponding welfare implications.

Proposition 3. *The equilibrium precision of buyer's information γ^* increases in her sensitivity to product differentiation t and decreases in her cost of information acquisition κ.*

Proposition 3 is immediate by differentiating the equilibrium condition (17) with respect to κ and t. It is also intuitive. The buyer's marginal benefit of information acquisition increases in t (as buying the "right" product affects the buyer's payoff more); whereas her marginal cost of information acquisition increases in κ; hence the equilibrium precision must increase in t and decrease in κ.

Proposition 4. *Suppose that v/t is sufficiently large, i.e., inequality* (10) *holds. As t increases, the equilibrium precision γ^* increases, and*

- *the seller's equilibrium price increases in the sense of first-order stochastic dominance, and the sellers' equilibrium profit $\pi = t\gamma^*(1+\sqrt{2})/2$ increases, and*
- *the buyer's equilibrium payoff $v + b(\gamma^*, \gamma^*) - \kappa c(\gamma^*)$ increases if $t < \beta_0/\beta_2$ and decreases if $t \geq \beta_0/\beta_2$ where β_0 and $\beta_2 > 0$ are specified in Lemma 4.*

Proposition 4 discuses the payoff consequence of increasing the buyer's sensitivity to product differentiation. As the buyer's sensitivity to product differentiation increases, the buyer chooses a higher equilibrium experiment precision according to Proposition 3. As a result, the competition between sellers is softened and the equilibrium price and seller's profit increases by Corollary 1. The impact of increasing t on consumer welfare is more complex and deserves some formal discussion. Differentiating the buyer's equilibrium payoff with respect to t yields

$$\Big[\underbrace{b_1(\gamma^*, \gamma^*) - \kappa c'(\gamma^*)}_{=0 \text{ by FOC}} + b_2(\gamma^*, \gamma^*)\Big]\frac{\partial \gamma^*}{\partial t} = (\beta_0 - \beta_2 t)\frac{\partial \gamma^*}{\partial t}. \tag{18}$$

Therefore, the impact of increasing t on the consumer welfare boils down to how it affects the sellers' *beliefs* about the buyer's information precision $\hat{\gamma}$ and the equilibrium price. As we have discussed, increasing sellers' belief about the buyer's experiment precision has two effects. By Lemma 4, the sign of b_2 is negative for $t > \beta_0/\beta_2$ and positive for $t \in (0, \beta_0/\beta_2]$. Therefore, the relationship between the buyer's welfare and her sensitivity to production differentiation is hump-shaped. In Figure 6, we plot the consumer's welfare u and the profit's profit π as functions of t for a given value of κ.

Proposition 5. *Suppose that v/t is sufficiently large, i.e., inequality* (10) *holds. As κ rises,*

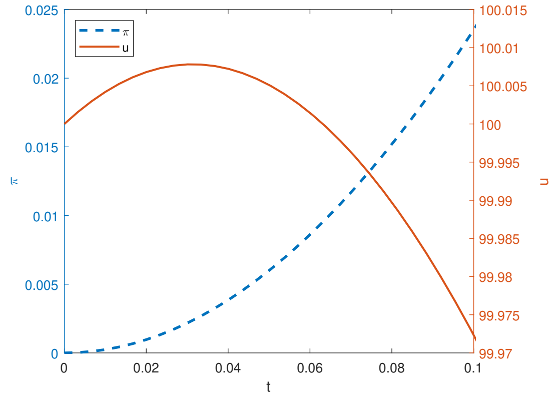


Figure 6: Consumer Surplus u and Seller Profit π over t. The cost function takes log-likelihood ratio form in equation (2), and parameter values are $v = 100$, $\kappa = 0.5$.

- *the sellers' equilibrium expected price falls, and the sellers' equilibrium profit* $\pi = t\gamma^*(1+\sqrt{2})/2$ *decreases, and*
- *the buyer's equilibrium payoff* $v + b(\gamma^*, \gamma^*) - \kappa c(\gamma^*)$ *is non-monotone.*

Proposition 5 discuses the welfare consequence of increasing the buyer's information acquisition marginal cost κ. As κ rises, the equilibrium precision falls according to Proposition 3, and the equilibrium price and seller's profit decreases by Corollary 1. Again, the impact of increasing κ on consumer welfare is more subtle. Differentiating the buyer's equilibrium payoff with respect to κ yields

$$\Big[\underbrace{b_1(\gamma^*,\gamma^*) - \kappa c'(\gamma^*)}_{=0 \text{ by FOC}} + b_2(\gamma^*,\gamma^*)\Big]\frac{\partial \gamma^*}{\partial \kappa} - c(\gamma^*) = (\beta_0 - \beta_2 t)\frac{\partial \gamma^*}{\partial \kappa} - c(\gamma^*). \tag{19}$$

The direct effect of increasing κ on the buyer's information acquisition cost is captured by $-c(\gamma^*)$ in expression (19), which is naturally negative. There is an indirect effect of increasing κ: it lowers the sellers' beliefs about the buyer's information precision. Again, this indirect effect can be either positive or negative, depending on the value of t and κ. Figure 7 simulates the buyer's payoff u and sellers' profit π as functions of κ for different values of t. The left panel corresponds to the case where t is small, and the buyer's welfare is monotonically increasing in κ. The right panel corresponds to the case where t is large. In this case, the buyer's payoff first decreases, then increases as κ goes up, exhibiting an U-shape.

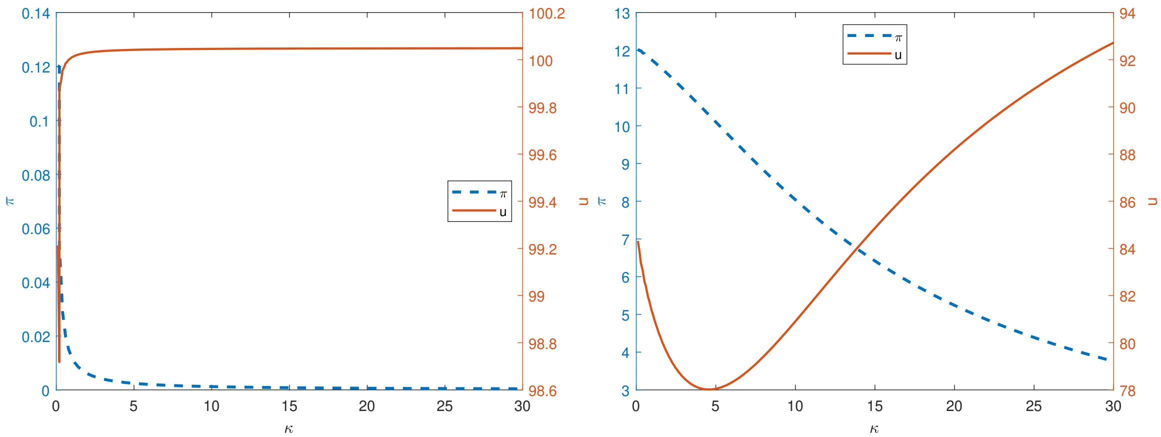


Figure 7: Consumer Surplus u and Seller's Profit π over κ. The cost function takes log-likelihood ratio form in equation (2), the parameter values are $v = 100$, $t = 0.1$ for the left panel, and $t = 10$ for the right panel.

4 An Application in Platforms

A major innovation in modern economies is where there is an intermediary between buyers and sellers. This has a long tradition from the general store, to department stores, such as Sears, up through the digital age where buyers purchase on a platform like Amazon and eBay. We can use our model to understand the trade-off that a platform faces when choosing how easy it is for buyers to gather information about the sellers that sell on the platform and which sellers it will want to be on the platform.

We now sketch a model that analyzes the platform's decisions. In addition to our base setup, we add a platform and now assume that there is a continuum of buyers of measure 1. Buyers are the same as described in the base model except that they have an exogenously given outside option that follows the continuous distribution $G(u_0)$ with associated density $g(u_0) > 0$ on the interval $[0, \bar{u}]$. A buyer's outside option is independent of her tastes, is a buyer's private information, and all buyers get i.i.d. draws. The platform gets an exogenous fraction $\alpha \in (0, 1)$ of a seller's revenue.

We examine two ways that the platform can influence the environment in which buyers and sellers interact: how differentiated the sellers are on the platform, the level of t, and how much easier the platform makes it for consumers to conduct an experiment to achieve a given level of precision, a reduction in κ, below κ_0. In the first case, one can view the platform as being dominant in the marketplace and having the ability to invite sellers by choosing a business model where sellers' products are either "standard" and do not provide consumers a large surplus above the base value, v, if it is a good match, or are "idiosyncratic" and consumers can get a large benefit above the base value from consumption if it is a good match. When the platform can reduce the cost parameter κ, one can view the platform as designing its website so as to to make it easier for a buyer to discover which seller is offering the best product for their tastes. This could include how easy it is for a buyer to learn what features would be most important to them and which product has those features by including space provided for the sellers to post details and from reviews from past buyers of the sellers' products. Also, one can interpret how carefully the platform strains "fake reviews" so as to make it cheaper for buyers to learn. Of course, by picking either t or κ, the platform is implicitly inducing a γ which is the equilibrium object of interest. We demonstrate that the analysis of the two settings are similar.

The timing differs from the base model in that there is an additional two stages added before the buyers choose the level of information and sellers choose prices. First, the platform chooses either t or κ. Next, buyers learn their outside option and decide whether to leave the market by taking their outside option. If they do not take the outside option, then the game proceeds as in the base model, while if they take the outside option they exit the game.

We will focus on the case v/t is sufficiently large, so that expression (10) holds. To construct the equilibrium of the model with an active platform, we use the results from the base model, Proposition 2, where for a given t there is induced a unique level of buyer precision γ and distribution of prices which generates an expected equilibrium level of utility for each buyer and expected seller profit. Define $u(t)$ as a buyer's expected utility if t is the choice of the platform. Then all consumers with an outside option less than $u(t)$ will join the platform, and the remaining $1 - G(u(t))$ consumers will exercise their outside option. The platform's payoff is increasing in both seller profits and consumer expected utility; it receives a fraction α of seller profits and the more consumers on the platforms the larger number of sales. By Proposition 3, higher t induces higher γ and more perceived product differentiation. From Proposition 4, higher γ raises sellers' expected prices and profits. Thus, just taking into account the seller channel, increasing t is always good for the platform. Using Proposition 4, the effect of a change of t on buyer payoffs is not monotone: payoffs rise in t for small t (below β_0/β_2) and fall in t, otherwise. Thus, the platform will never choose a $t < \beta_0/\beta_2$. But, as it raises t above β_0/β_2, then this will reduce the number of users on the platform. This introduces a trade-off for the platform, since while the profit per user is rising, the number of users is falling as t increases.

Thus, the platform's problem when choosing t is

$$\max_{t \geq \beta_0/\beta_2} \alpha G(u(t))[t\gamma(t)(1+\sqrt{2})] \tag{20}$$

The first order condition with respect to t is

$$\alpha g(u(t))\frac{du}{dt}[t\gamma(t)(1+\sqrt{2})] + \alpha G(u(t))(1+\sqrt{2}) + \alpha G(u(t))\frac{d\gamma}{dt}t(1+\sqrt{2}) = 0. \tag{21}$$

The first term shows the loss in users as t is raised above β_0/β_2; it consists of the measure of lost consumers, $g(u(t))\frac{du}{dt}$ times the amount of lost revenue from each of these consumers, $\alpha[t\gamma(t)(1+\sqrt{2})]$. The second term represents the direct effect of an increase in t on platform profits by increasing the sellers' prices. The final term is the indirect effect that has an increase in t raising buyers' equilibrium experiment precision times the resulting increase in profit per consumer for a given level of users. Thus, the platform is facing a more subtle version of the classic, trade-off that sellers face when raising price and losing demand. Now, raising t raises price via two channels, the direct on price and the indirect one via γ balanced with the loss of sales.

Now, assume that the platform can choose how much easier it makes it for a consumer to achieve a given level of precision of their experiments below the initial level of κ_0. Again, we examine the setting when v/t is large. Now, instead of utilities and profits being a function of t they are functions of κ. As can be seen from Proposition 3, the equilibrium γ is falling in κ. By Proposition 5, seller profits are always falling in κ, since the equilibrium precision is falling in it. The effect of an increase in κ on buyer utility is less clear cut; see Proposition 5 and Figure 7. In the first panel of Figure 7, when t is small, buyer surplus is everywhere increasing in κ, while in the second, when t is large, utility is initially falling in κ and then increasing.

The platform's objective function in κ is

$$\max_{\kappa \in [0,\kappa_0]} \alpha G(u(\kappa))[t\gamma(\kappa)(1+\sqrt{2})] \tag{22}$$

The first order condition with respect to κ is

$$\alpha g(u(\kappa))\frac{du}{d\kappa}[t\gamma(\kappa)(1+\sqrt{2})] + \alpha G(u(\kappa))\frac{d\gamma}{d\kappa}t(1+\sqrt{2}). \tag{23}$$

The second term, which represents how seller revenue changes in κ with a given measure of consumers, is always negative since γ is falling in κ. If the first term is always increasing in

κ as in the first panel of Figure 7 when t is small, then if the first order condition is satisfied at some value below κ_0, then the platform will choose that level to reduce the cost of consumers doing an experiment so as to expand the market; it balances price with consumer welfare (quantity). If the first order condition is never positive, then the platform makes experimentation as cheap as possible and maximizes price. If the first term is not everywhere increasing, and is initially negative then turns positive as in the second panel of Figure 7, when t is large, the platform compares its profit level with the smallest possible $\kappa = 0$ with κ_0 and chooses the one that maximizes its profits.

5 Conclusion

We offer a new approach to consumer interaction with firms where the degree of perceived product differentiation is endogenous. This allowed us to arrive at some interesting comparative statics. We also offer a rudimentary example of how a profit maximizing platform would view the trade-offs that it would face when designing the environment that consumers and firms will interact on. We think that this is a first step in viewing the very complicated situation that modern platforms face. A more elaborate model would allow for the possibility for the platform to be one of the sellers.

There are many possible avenues for interesting research in the future, for example, allowing for more than two firms and seeing the effect of additional competition. Furthermore, if consumers have a natural inclination towards one firm, it is the default firm. This would lead to both asymmetric pricing distributions and change a consumer's experimentation level. This is will be especially relevant in the platform context with self referencing. Finally, for some applications, assuming that consumers observe prices and then make information acquisition decisions accordingly is more realistic. In this case, firms will be incentivized to set prices to influence consumers' information acquisition favoring their products.

A Appendix: An Equilibrium with Atom of the Pricing Game

Lemma 5. *Suppose that $\gamma > 0$ and*

$$t(3\gamma - 1) < 2v < t\left[(3 + 2\sqrt{2})\gamma - 1\right]. \tag{24}$$

The pricing game has a symmetric Nash equilibrium which is symmetric and in mixed-strategy. In equilibrium, sellers randomize over $[\hat{p} - t\gamma, \check{p}]$ according to CDF

$$\hat{F}(p) = \begin{cases} 1 - \frac{2\hat{\pi}}{p + t\gamma} & \text{if } p \in [\hat{p} - t\gamma, \check{p} - t\gamma) \\ 1 - \frac{2\hat{\pi}}{\check{p}} & \text{if } p \in [\check{p} - t\gamma, \hat{p}) \\ 2 - \frac{2\hat{\pi}}{p - t\gamma} & \text{if } p \in [\hat{p}, \check{p}) \\ 1 & \text{if } p = \check{p} \end{cases},$$

where $\check{p} = v + t(1+\gamma)/2$, $\hat{p} = \left[t\gamma + \sqrt{(t\gamma)^2 + 4t\gamma\check{p}}\right]/2$, and the seller's equilibrium profit is

$$\hat{\pi} = \frac{\hat{p}}{2} = \frac{t\gamma + \sqrt{(t\gamma)^2 + 4t\gamma\check{p}}}{4} < \pi,$$

where π has been specified in Lemma 2.

Proof of Lemma 5. Given our construction, a seller will never play any price in $[\check{p}, \hat{p})$, so $F(\check{p}) = F(\hat{p})$, which implies that $1 - \frac{\hat{p}}{\check{p}} = 2 - \frac{\hat{p}}{\hat{p} - t\gamma}$, or $\hat{p} = \frac{t\gamma + \sqrt{(t\gamma)^2 + 4t\gamma\check{p}}}{2}$, and the equilibrium profit is $\pi = \frac{t\gamma + \sqrt{(t\gamma)^2 + 4t\gamma\check{p}}}{4}$.

What remains is to verify that F is an equilibrium, since the uniqueness argument is identical to the one of Lemma 2. First, we verify that sellers are indifferent to choose any price in the equilibrium support $[\hat{p} - t\gamma, \check{p} - t\gamma) \cup [\hat{p}, \check{p}]$. There are two cases to consider. If a seller chooses any price $p \in [\hat{p}, \check{p}]$, his payoff is

$$\int_{p - t\gamma}^{\overline{p}} \frac{1}{2} p dF(\tilde{p}) = \frac{1}{2} p \left[1 - F(p - t\gamma)\right] = \frac{1}{2}\hat{p} = \hat{\pi}$$

If a seller chooses any price $p \in [\hat{p} - t\gamma, \check{p} - t\gamma)$, his payoff is

$$\int_{\hat{p} - t\gamma}^{p + t\gamma} \frac{1}{2} p dF(\tilde{p}) + \int_{p + t\gamma}^{\check{p}} p dF(\tilde{p}) = p\left[1 - \frac{1}{2}F(p + t\gamma)\right] = \frac{1}{2}\hat{p} = \hat{\pi}.$$

Third, we verify that there is no profitable deviation. Since $\check{p}$ is the buyer's maximum willingness to pay for any product, it is sufficient to rule out deviation to prices in $[0, \hat{p} - t\gamma) \cup (\check{p} - t\gamma, \hat{p})$.

- Suppose the seller deviates to a price $p < \hat{p} - t\gamma$; his profit is

$$\pi_b(p) = \begin{cases} p & \text{if } p \le \hat{p} - 2t\gamma \\ \frac{p}{2}[2 - 1 + \frac{2\pi}{p+2t\gamma}] & \text{if } p \in (\hat{p} - 2t\gamma, \check{p} - 2t\gamma) \\ \frac{p}{2}[2 - 1 + \frac{2\pi}{\check{p}}] & \text{if } p \in (\check{p} - 2t\gamma, \hat{p} - t\gamma) \end{cases},$$

  where $\frac{p}{2}[2 - F(p + t\gamma)]$ is increasing in p as its first order derivative for $p \in (\hat{p} - 2t\gamma, \overline{p} - 2t\gamma)$

$$\begin{aligned} & \frac{1}{2}\left[2 - F\left(p + t\gamma\right)\right] - \frac{p}{2} f\left(p + t\gamma\right) \\ = & \frac{1}{2}\left[2 - 1 + \frac{2\pi}{p + 2t\gamma}\right] - \frac{p}{2}\frac{2\pi}{\left(p + 2t\gamma\right)^2} \\ = & \frac{2\pi t\gamma}{\left(p + 2t\gamma\right)^2} + \frac{1}{2} > 0 \end{aligned}$$

  so $\pi_b\left(p\right)$ achieves its maximum at $\hat{p} - t\gamma$, which is $\frac{\hat{p} - t\gamma}{2}\left(1 + \frac{2\pi}{\overline{p}}\right) = \hat{\pi}$ as

$$\frac{\hat{p} - t\gamma}{2}\left(1 + \frac{2\pi}{\check{p}}\right) - \hat{\pi} = \frac{\hat{p} - t\gamma}{2}\left(1 + \frac{2\pi}{\check{p}}\right) - \frac{\hat{p}}{2} = 0;$$

  hence, $\pi_b\left(p\right) < \pi$ for any $p < \hat{p} - t\gamma$.

- Suppose the seller deviates to a price $p \in (\check{p} - t\gamma, \hat{p})$. Then the seller always gets a half market and its deviation profit is $\pi_m\left(p\right) = \frac{1}{2}p$. The maximum deviation profit is $\frac{1}{2}\hat{p} = \hat{\pi}$. Hence, $\pi_m\left(p\right) < \pi$ for any $p \in (\check{p} - t\gamma, \hat{p})$.

In sum, there is no profitable deviation to $p < \hat{p} - t\gamma$ or $p \in (\check{p} - t\gamma, \hat{p})$.

□

In the equilibrium described by Lemma 5, sellers play $p = v + (1 - \gamma)/2$ with a strictly positive probability $1 - F(\check{p}^-)$ and randomize over $[\hat{p}, \check{p}) \cup [\hat{p} - t\gamma, \check{p} - t\gamma)$. To see why this is an equilibrium, a seller believes that his rival will play $\check{p}$ with a probability less than one, which mitigates his incentive to cut price by $t\gamma$ to steal business. As in Lemma 2, a seller is indifferent between any price in $[\hat{p}, \check{p})$ due to the construction of $F(\cdot)$ on $[\hat{p} - t\gamma, \check{p} - t\gamma)$, and

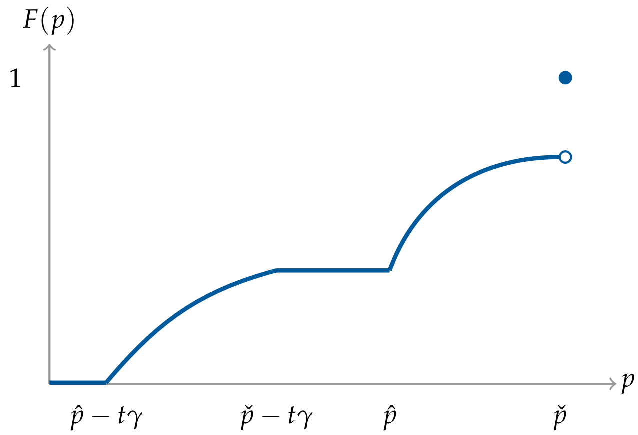


Figure 8: Mixed-Strategy Equilibrium II CDF.

any price in $[\hat{p} - t\gamma, \check{p} - t\gamma)$ due to the construction of $F(\cdot)$ on $[\hat{p}, \check{p})$. When a seller changes the price in these two intervals, the corresponding change in demand exactly offsets the impact on his profit. The balance between these two effects will be broken for any $p < \hat{p} - t\gamma$ or $p > \check{p}$ as in Lemma 2. A remarkable feature of this equilibrium is that there is a "gap" in the equilibrium support. To see why, notice that a seller's profit is discontinuous at $\check{p} - t\gamma$ because F has an atom at $\check{p}$. Consider the event that seller j chooses $\check{p}$, which will take place with a strictly positive probability. If the seller chooses a price arbitrarily close but at least $p - t\gamma$, he splits the market with seller j and gets a profit $p/2$. If seller $i \neq j$ chooses a price strictly less but arbitrarily close to $\check{p} - t\gamma$, he gets a discrete increase in profit above $p/2$. Hence, it must be strictly suboptimal to choose any price $p \in [\check{p} - t\gamma, \hat{p})$. See Figure 9 for a visualized illustration.

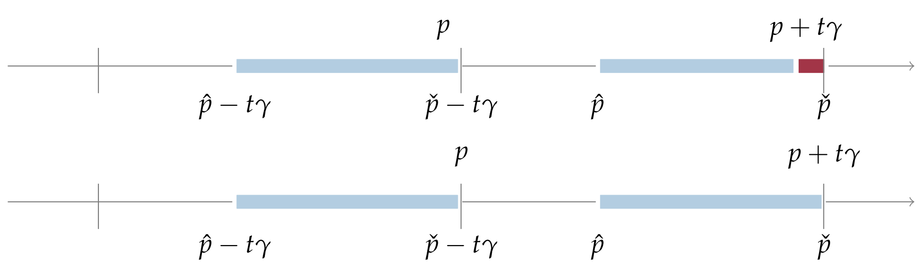


Figure 9: In the upper panel, a seller chooses a price p that is below but close to $\check{p} - t\gamma$. His demand is 0.5 if his rival's price is less than or equal to $p + t\gamma$, and it is 1 otherwise. As $p \nearrow \check{p} - t\gamma$, the demand converges to $0.5 + 1 - \lim_{p \nearrow \check{p} - t\gamma} F(p) > 0.5$. In the lower panel, a seller chooses the price to be exactly $\check{p} - t\gamma$. His demand is 0.5 for any equilibrium price that his rival plays.

The result in Lemma 5 depicts an intermediate case between Lemmas 1 and 2. As v and t change, the equilibrium profit and price (range) respond monotonically and "continuously." Formally,

Corollary 2. *Suppose that $\gamma > 0$ and condition (24) holds. Then $\hat{\pi}, \hat{p}$ increase in v and t, and $\check{p}$ increases in v. Moreover,*

- *as $2v \to t\left[(3+2\sqrt{2})\gamma - 1\right]$, the equilibrium distribution and sellers' profit converge to the ones specified in Lemma 2, i.e., $\hat{p} \to t\gamma(1+\sqrt{2}), \check{p} \to \bar{p}$, $\hat{F}(v+0.5t(1+\gamma)) \to 1$, and $\hat{\pi} \to \pi$ where $\bar{p}, \pi$ have been specified in Lemma 2, and*
- *as $2v \to t(3\gamma - 1)$, the equilibrium strategy and sellers' profit converge to the ones specified in Lemma 1 case 2, i.e., $\hat{p} \to \check{p}, \hat{\pi} \to 0.5\check{p}$.*

The proof is by straightforward algebra, which is omitted.

B Appendix: Omitted Proofs

Proof of Corollary 1. Using the equilibrium distribution in (8) and the expressions of $\bar{p}$ and $\underline{p}$ in Lemma 2, the expected price can be expressed as

$$\begin{aligned}\int_{\underline{p}}^{\bar{p}} pdF\left(p\right) =& t\gamma(1+\sqrt{2})\Big[-\int_{\underline{p}}^{\underline{p}+t\gamma} pd\frac{1}{p+t\gamma} - \int_{\underline{p}+t\gamma}^{\bar{p}} pd\frac{1}{p-t\gamma}\Big] \\ =& t\gamma[\underbrace{(1+\sqrt{2})\ln(1+\sqrt{2})+\sqrt{2}-1}_{A}],\end{aligned}$$

which increases in γ. The variance is

$$\begin{aligned}\int_{\underline{p}}^{\bar{p}}(p-At\gamma)^2dF(p) &= (\bar{p}-At\gamma)^2 + 2At\gamma\int_{\underline{p}}^{\bar{p}} F(p)dp - 2\int_{\underline{p}}^{\bar{p}} pF(p)dp \\ &= (\bar{p}-At\gamma)^2 + 2At\gamma(\bar{p}-At\gamma) - 2(t\gamma)^2B\end{aligned}$$

where

$$B = \tfrac{3}{2}(1+\sqrt{2})^2 - 1 - 2(1+\sqrt{2}) - (1+\sqrt{2})\ln\tfrac{1+\sqrt{2}}{2}.$$

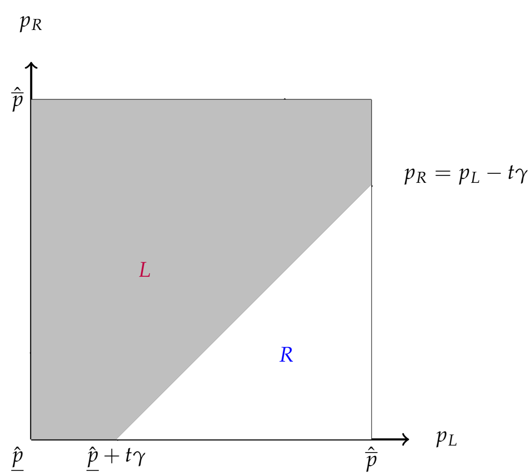
We can further simplify the expression of the variance and get $C(t\gamma)^2$ where $C = -A^2 - (1+\sqrt{2})^2 + 2 + 4(1+\sqrt{2}) + 2(1+\sqrt{2})\ln\frac{1+\sqrt{2}}{2} > 0$, so the variance is increasing in γ.

□

Proof of Lemma 4. The buyer expects that sellers set the symmetric mixed pricing strategy by $\hat{F} \equiv F(\cdot|\hat{\gamma})$ over $[\underline{\hat{p}}, \hat{\overline{p}}]$. Hence, the buyer's expected payoff of choosing information precision γ given sellers' strategy $\hat{F}$ is

$$\begin{aligned} b(\gamma, \hat{\gamma}) &= v + \frac{t}{2} - \underbrace{\int_{\underline{\hat{p}}}^{\hat{\overline{p}}} \int_{\underline{\hat{p}}}^{\min\{p'+t\gamma, \overline{p}\}} \left(p - \frac{t\gamma}{2}\right) d\hat{F}(p) d\hat{F}(p')}_{\text{area L: obedience}} \\ &\quad - \underbrace{\int_{\underline{\hat{p}}}^{\hat{\overline{p}}} \int_{p'+t\gamma}^{\hat{\overline{p}}} \left(p' + \frac{t\gamma}{2}\right) d\hat{F}(p) d\hat{F}(p')}_{\text{area R: dis-obedience}}. \end{aligned} \tag{25}$$

The price region $\{(p_L, p_R) : (p_L, p_R) \in [\underline{\hat{p}}, \hat{\overline{p}}]^2\}$ is divided by $p_R = p_L - t\gamma$, which is visualized in the following figure. In the area L of $p_R > p_L - t\gamma$, the buyer purchases obediently, while the buyer purchases dis-obediently in the area R of $p_R < p_L - t\gamma$.



We are interested in the partial derivatives of function $b(\gamma, \hat{\gamma})$ at $\gamma = \hat{\gamma}$. When $\gamma \geq 2\hat{\gamma}$, the buyer always purchases obediently (area R disappears). When $\gamma < 2\hat{\gamma}$, the obedience part of equation (25) is

$$\begin{aligned} &\int_{\underline{\hat{p}}}^{\hat{\overline{p}}} \int_{\underline{\hat{p}}}^{\min\{p'+t\gamma, \overline{p}\}} \left(p - \frac{t\gamma}{2}\right) d\hat{F}(p) \, d\hat{F}(p') \\ = &\int_{\underline{\hat{p}}}^{\underline{\hat{p}}+t\gamma} \left(p - \frac{t\gamma}{2}\right) d\hat{F}(p) + \int_{\underline{\hat{p}}+t\gamma}^{\hat{\overline{p}}} \left(p - \frac{t\gamma}{2}\right) \left[1 - \hat{F}(p - t\gamma)\right] d\hat{F}(p), \end{aligned} \tag{26}$$

and the disobedience part of equation (25) is

$$\int_{\underline{\hat{p}}}^{\hat{\bar{p}}} \int_{p'+t\gamma}^{\hat{\bar{p}}} \left(p' + \frac{t\gamma}{2}\right) d\hat{F}(p)\, d\hat{F}(p') = \int_{\underline{\hat{p}}}^{\hat{\bar{p}}-t\gamma} \left(p' + \frac{t\gamma}{2}\right) \left[1 - \hat{F}(p' + t\gamma)\right] d\hat{F}(p'). \quad (27)$$

Plugging equations (26) and (27) into the right hand side of equation (25) yields

$$\begin{aligned} b(\gamma, \hat{\gamma}) &= v + \frac{t}{2} - \int_{\underline{\hat{p}}}^{\underline{\hat{p}}+t\gamma} \left(p - \frac{t\gamma}{2}\right) d\hat{F}(p) - \int_{\underline{\hat{p}}+t\gamma}^{\hat{\bar{p}}} \left(p - \frac{t\gamma}{2}\right) \left[1 - \hat{F}(p - t\gamma)\right] d\hat{F}(p) \\ &\quad - \int_{\underline{\hat{p}}}^{\hat{\bar{p}}-t\gamma} \left(p' + \frac{t\gamma}{2}\right) \left[1 - \hat{F}(p' + t\gamma)\right] d\hat{F}(p'). \end{aligned} \quad (28)$$

What remains is to simplify the partial derivatives of b evaluated at $\gamma = \hat{\gamma}$. We give the sketch here and leave tedious algebra to the online Supplementary Appendix.

Step 1. We show that $b_1(\gamma, \gamma) = \beta_1 t$ with $\beta_1 > 0$. By expression (28), we have

$$\begin{aligned} b_1(\gamma, \hat{\gamma}) &= \tfrac{t}{2}\hat{F}(\underline{\hat{p}} + t\gamma) - \int_{\underline{\hat{p}}}^{\hat{\bar{p}}-t\gamma} \left[t\left(p + \tfrac{t\gamma}{2}\right)\hat{f}(p) - \tfrac{t}{2}\left(1 - \hat{F}(p)\right)\right] d\hat{F}(p) \\ &\quad + \int_{\underline{\hat{p}}+t\gamma}^{\hat{\bar{p}}} \left[t\left(p - \tfrac{t\gamma}{2}\right)\hat{f}(p) - \tfrac{t}{2}\left(1 - \hat{F}(p)\right)\right] d\hat{F}(p) \end{aligned} \quad (29)$$

Then we plug the sellers' best response $\hat{F}$ into equation (29) and get

$$\begin{aligned} b_1(\gamma, \hat{\gamma}) &= \tfrac{t}{2}\left[2 - \tfrac{t\hat{\gamma}(1+\sqrt{2})}{(\sqrt{2}-1)t\hat{\gamma}+t\gamma}\right] - \int_{\underline{\hat{p}}}^{\hat{\bar{p}}-t\gamma} \left[t\left(p + \tfrac{t\gamma}{2}\right)\tfrac{t\hat{\gamma}(1+\sqrt{2})}{(p+t\hat{\gamma})^2} - \tfrac{t}{2}\tfrac{t\hat{\gamma}(1+\sqrt{2})}{p+t\hat{\gamma}}\right]\tfrac{t\hat{\gamma}(1+\sqrt{2})}{(p+t\hat{\gamma})^2}dp \\ &\quad + \int_{\underline{\hat{p}}+t\gamma}^{\hat{\bar{p}}} \left[t(p - \tfrac{t\gamma}{2})\tfrac{t\hat{\gamma}(1+\sqrt{2})}{(p-t\hat{\gamma})^2} - \tfrac{t}{2}(\tfrac{t\hat{\gamma}(1+\sqrt{2})}{p-t\hat{\gamma}} - 1)\right]\tfrac{t\hat{\gamma}(1+\sqrt{2})}{(p-t\hat{\gamma})^2}dp \\ &= \tfrac{t}{2}\left[2\sqrt{2} - 1 + \tfrac{(3+2\sqrt{2})t\gamma}{(\sqrt{2}-1)t\hat{\gamma}+t\gamma}\right] + \tfrac{t}{2}\left[\tfrac{1}{2}(\tfrac{(\hat{\bar{p}}-t\hat{\gamma})^2}{(\hat{\bar{p}}-t\gamma+t\hat{\gamma})^2} - 1) + \tfrac{t\gamma-2t\hat{\gamma}}{3}(\frac{(\hat{\bar{p}} - t\hat{\gamma})^2}{(\hat{\bar{p}} - t\gamma + t\hat{\gamma})^3} - \tfrac{1}{\underline{\hat{p}}+t\hat{\gamma}})\right] \\ &\quad + \frac{t}{2}\left[\tfrac{1}{2}(\tfrac{(\hat{\bar{p}}-t\hat{\gamma})^2}{(\underline{\hat{p}}+t\gamma-t\hat{\gamma})^2} - 1) + \tfrac{t\gamma-2t\hat{\gamma}}{3}(\frac{1}{\hat{\bar{p}} - t\hat{\gamma}} - \frac{(\hat{\bar{p}}-t\hat{\gamma})^2}{(\underline{\hat{p}}+t\gamma-t\hat{\gamma})^3})\right] + \tfrac{t}{2}\left(\tfrac{\hat{\bar{p}}-t\hat{\gamma}}{\underline{\hat{p}}+t\gamma-t\hat{\gamma}} - 1\right). \end{aligned}$$

Evaluating the above expression at $\hat{\gamma} = \gamma \in (0, 1)$ yields

$$b_1(\gamma, \gamma) = \frac{t}{2}\left[2\sqrt{2} - 1 + \tfrac{(3+2\sqrt{2})t\gamma}{\sqrt{2}t\gamma}\right] = \tfrac{1}{2}\left[\tfrac{9\sqrt{2}}{2} + 1 + \tfrac{(1+\sqrt{2})^2}{6\sqrt{2}}(1 - \tfrac{1}{(1+\sqrt{2})^3})\right]t = \beta_1 t > 0$$

Step 2. We show that $b_2(\gamma, \gamma) = -\beta_2 t + \beta_0$. Letting $\hat{\gamma} = \gamma$ in equation (28) yields

$$
\begin{aligned}
b_2(\gamma,\gamma) &= (\underline{p}-\tfrac{t\gamma}{2})f(\underline{p})\sqrt{2}t-\int_{\underline{p}}^{\underline{p}+t\gamma}(p-\frac{t\gamma}{2})f'(p)dp \\
&\quad -(\overline{p}-\tfrac{t\gamma}{2})\Big[1-F(\overline{p}-t\gamma)\Big]f(\overline{p})(2+\sqrt{2})t \\
&\quad -\int_{\underline{p}+t\gamma}^{\overline{p}}(p-\frac{t\gamma}{2})\Big[(1-F(p-t\gamma))f'(p)-f(p-t\gamma)f(p)\Big]dp \\
&\quad +(\underline{p}+\tfrac{t\gamma}{2})\Big[1-F(\underline{p}+t\gamma)\Big]f(\underline{p})\sqrt{2}t \\
&\quad -\int_{\underline{p}}^{\overline{p}-t\gamma}(p+\tfrac{t\gamma}{2})\left[(1-F(p+t\gamma))f'(p)-f(p+t\gamma)f(p)\right]dp \\
&= \beta_2 t+\beta_0,
\end{aligned}
$$

where $\beta_2 = -\frac{1+2\sqrt{2}}{1+\sqrt{2}}$ and $\beta_0 = 7/2$.

□

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