# Propaganda, Alternative Media, and Accountability in Fragile Democracies

Anqi Li[1], Davin Raiha[2], and Kenneth W. Shotts[3]

[1]Department of Economics, Washington University in St. Louis. anqili@wustl.edu.
[2]Kelley School of Business, Indiana University. draiha@iu.edu.
[3]Stanford Graduate School of Business. kshotts@stanford.edu.

**Abstract**

We develop a model of electoral accountability with mainstream and alternative media. In addition to regular high- and low-competence types, the incumbent may be an aspiring autocrat who controls the mainstream media and will subvert democracy if retained in office. A truthful alternative media can help voters identify and remove these subversive types while re-electing competent leaders. A malicious alternative media, in contrast, spreads false accusations about the incumbent and demotivates policy effort. If the alternative media is very likely be malicious and hence is unreliable, voters ignore it and use only the mainstream media to hold regular incumbents accountable, leaving aspiring autocrats to win re-election via propaganda that portrays them as effective policymakers. When the alternative media's reliability is intermediate, voters heed its warnings about subversive incumbents, but the prospect of being falsely accused demotivates effort by regular incumbents and electoral accountability breaks down.

Keywords: propaganda, alternative media, electoral accountability and selection, fragile democracy

Many countries inhabit a grey area between democracy and autocracy: their leaders are elected, but try to eliminate checks on their power and subvert the institutional foundations of democracy. One check that aspiring autocrats often remove is the mainstream media, which can be induced, by a combination of censorship, ownership, and corruption, to refrain from criticism and act as a propaganda vehicle for the regime. Recent rulers who have taken this approach include Turkey's Erdoğan, Hungary's Orban, and Venezuela's Chávez.

Citizens who are unsure about their leaders and skeptical of the mainstream media may turn to alternative media.[1] For our analysis, the alternative media has two defining characteristics. First, it is independent of the government and beyond its control. Second, citizens are unsure about its intentions, which can range from providing truthful warnings about aspiring autocrats to making malicious accusations against legitimate leaders. The first kind of intention was demonstrated by Peru's Canal N, which played a key role in the downfall of President Fujimori. The second kind of intention was demonstrated by right-wing commentators in the United States, who accused President Obama of many things, including being a Marxist and, in the words of InfoWars's Alex Jones, a "would-be dictator." Other examples of alternative media that are difficult for the government to control include opposition television and newspapers; foreign news providers; and, more recently, social media platforms that contain a mixture of accurate information, conspiracy theories, and disinformation. A key characteristic of all these examples is that they can be either truthful or malicious and their intentions are uncertain from (at least) some voters' perspectives.

We develop a model of electoral selection and accountability, in which citizens are uncertain about the incumbent's type, the mainstream media's independence, and the alternative media's intentions. Voters want to incentivize and retain competent leaders while removing those who are incompetent or autocratic. When the mainstream media praises the incumbent, voters don't know whether this is neutral praise of skillful policymaking or propaganda for an aspiring autocrat. The alternative media can be either a truthful type that provides accurate warnings about autocratic

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[1]Though the terms "alternative media" and "independent media" are often used interchangeably, we will use "alternative media" throughout the paper.

subversion and propaganda, or it can be a malicious type that makes false accusations.

We study how the alternative media affects accountability and selection. A key parameter in our analysis is the alternative media's reliability, defined as the probability that it is truthful. A highly-reliable alternative media is beneficial, because it helps citizens remove aspiring autocrats while holding regular non-autocratic politicians accountable for their policymaking. On the other hand, if the alternative media is highly unreliable, voters expect it to "cry wolf," so they ignore its warnings. They use the mainstream media to hold regular incumbents accountable, leaving autocratic types to win re-election via propaganda and subsequently subvert democracy.

A more subtle effect occurs when the alternative media is too reliable to be ignored by voters, yet sufficiently unreliable that competent incumbents worry about being falsely accused. In that case, accountability breaks down and selection is based only on the alternative media's report.

Our model has implications for two important features of contemporary politics: fake news and democratic backsliding. Our model shows that fake news doesn't just induce voters to make mistakes. Rather, false accusations made by somewhat-reliable alternative media sources can also demotivate incumbent effort and thereby undermine electoral accountability.

In the context of fragile democracies, our theory suggests a tension between accountability for policymaking and prevention of democratic backsliding. In many circumstances, elections cannot simultaneously achieve both of these important goals, and voters who seek to re-elect effective leaders may fail to heed warning signs that the incumbent is an aspiring autocrat.

Our theory sits at the intersection of the literatures on media bias, electoral accountability, and fragile democracies. The media's role in accountability has been examined by many scholars, including some who analyze pro- or anti-incumbent biases (Ashworth and Shotts 2010, Warren 2012, Wolton 2019). While existing models (e.g., Besley and Prat 2006) analyze capture of mainstream media, we study the effect of alternative media that is independent of government control and either truthfully reveals or falsely accuses the incumbent of being a subversive autocrat—a feature that is absent from existing models.

The literature on autocracy and fragile democracy includes many theories of propaganda and censorship (Egorov, Guriev, and Sonin 2009; Gehlbach and Sonin 2014; Lorentzen 2014; Cheah 2016; Little 2017; Horz 2018). There is a growing formal literature on democratic backsliding (e.g., Luo and Przeworski 2019, Svolik 2020), but very few scholars analyze accountability in weak democracies (though see Svolik 2013). Also, with the exception of Nalepa, Vanberg, and Chiopris (2018) most models of backsliding don't incorporate uncertainty about whether the incumbent is a subversive autocrat. Our setting is closest to Guriev and Treisman's (2018) model of propaganda and censorship by a regime that wishes to appear competent. However, in their model citizens are only concerned about whether the ruler is competent, and criticism of the incumbent is always accurate, whereas a key parameter in our analysis is the reliability of the alternative media.

## Model Setup

Consider a model of policymaking and elections, with five actors: the incumbent, challenger, mainstream media, alternative media, and voter. There are two equally-likely states of the world, $\omega \in \{0, 1\}$. The incumbent may learn $\omega$ and chooses a policy $x \in \{0, 1\}$, where the correct policy for the voter's interest is the one that matches the state. Then the mainstream media announces a message $m \in \{0, 1\}$ about the policy it believes is correct. The alternative media simultaneously issues a report $r \in \{S, NS\}$ about whether the incumbent is a subversive type who uses the mainstream media for propaganda. Finally, the voter observes the policy choice $x$ and the mainstream and alternative media messages $m$ and $r$, but not the true state $\omega$, and decides between the incumbent and challenger.

The incumbent can be a high, low, or subversive type, $\theta_I \in \{H, L, S\}$. High- and low-competence types are standard in the accountability literature. The subversive type is novel: she controls the mainstream media and if re-elected will consolidate her power and deliver a negative payoff to the voter. The incumbent is subversive with probability $\sigma$. Conditional on not being subversive, she is a high type with probability $\pi$. Low and subversive types only know the prior $\Pr(\omega = 1) = \frac{1}{2}$ and

3

cannot acquire additional information. A high type can exert effort, at cost $k$, to learn the true state before choosing policy. The winner of the election gets an ego-rent of 1.

The mainstream media is either truthful or propagandist. If truthful, it non-strategically issues a report $m \in \{0, 1\}$ that matches the true state $\omega$ with probability $q \in \left(\frac{1}{2}, 1\right)$. The mainstream media is a propagandist if and only if the incumbent is subversive, in which case it always reports that the incumbent's policy choice was correct, $m = x$.

The alternative media is either truthful or malicious. If truthful, it reports that the incumbent is subversive and is using the mainstream media for propaganda, $r = S$, if and only if this is indeed the case. If malicious, it always reports $r = S$.[2] A central parameter in our analysis is the probability that it is malicious, $\phi \in [0, 1]$. Hereafter $1 - \phi$ is referred to as the alternative media's *reliability*.

After observing $x$, $m$ and $r$, the voter elects the incumbent or the challenger. She gets utility 1 from re-electing a high-type incumbent, 0 from a low type, and $-s$ from a subversive type.[3] As is standard in accountability models, we assume that the challenger is inactive until he is elected, in which case he delivers an exogenous expected utility $U_C \in [-s, 1]$ to the voter.

We characterize Perfect Bayesian equilibria that are symmetric with respect to policies 0 and 1, which means low- and subversive-type incumbents choose each policy with probability $\frac{1}{2}$. Other equilibrium components are: (1) the high-type incumbent's effort decision and (2) the voter's belief about the incumbent, as well as his election decision. In the paper we focus on intuition; statements of equilibria can be found in the online appendix.[4]

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[2]Malice could be driven by either supply side factors (e.g., an owner who wants to discredit the incumbent and the mainstream media) or demand side factors (e.g., catering to a subset of the population that despises the incumbent). Our results hold regardless of the source of malice.

[3]A possible objection is that some autocrats turn out to be effective policymakers who are well-liked by their subjects. In our model, $-s$ represents voters' *expected* payoff from retaining a subversive type. This can include autocrats who turn out to be "good" along with those who turn out to be disastrous, as long as the expected payoff is negative. We also note that even if the autocratic type could exert high effort at a cost, she would have no incentive to do so, because she can induce the mainstream media to praise her policy decisions.

[4]In the online appendix, we also show that our main results hold qualitatively in an extended model, in which the subversive type decides whether to capture the mainstream media and also has some ability to influence the alternative media. Possible topics for future extensions include allowing voters to sometimes directly learn $\omega$, incorporating politicians with biased policy preferences, and making the model dynamic, but a comprehensive analysis of such model variants is beyond the scope of the current paper.

## Baseline

To establish a baseline, suppose there is no alternative media. We say there is *accountability* if two conditions hold: (1) high-type incumbents exert effort to choose policy in the voter's interest and (2) the incumbent is removed from office if $m \neq x$. These conditions are mutually reinforcing: a high type exerts effort to choose good policies and earn praise from the mainstream media, and the fact that she exerts effort means the media message $m$ is informative about the incumbent's type. We analyze whether it is possible to have accountability in equilibrium, focusing on the incumbent's incentives to exert effort and the voter's use of information provided by the media.

The voter's inferences about the incumbent are complicated by the possibility of propaganda. If propaganda is very likely, the mainstream media message is essentially meaningless. If propaganda isn't very likely, a message that the incumbent chose the correct policy ($m = x$) conveys positive information about the incumbent's desirability. Assuming accountability, high-type incumbents choose $x = \omega$ and low and subversive types choose each policy with probability $\frac{1}{2}$, so the voter's expected utility from re-electing the incumbent is:

$$\overline{U} := 1 \cdot \Pr\left(\theta_I = H | m = x\right) - s \cdot \Pr\left(\theta_I = S | m = x\right) = \frac{\pi q - sl}{\pi q + (1 - \pi)\frac{1}{2} + l},$$

where $l := \frac{\sigma}{1-\sigma}$ is the likelihood that the incumbent is subversive. Meanwhile, when $m \neq x$, the voter knows the incumbent isn't subversive, and his utility from re-electing her is:

$$\underline{U} := 1 \cdot \Pr\left(\theta_I = H | m \neq x\right) = \frac{\pi (1 - q)}{\pi (1 - q) + (1 - \pi)\frac{1}{2}}.$$

Accountability thus requires two conditions. First, the incumbent and challenger must be similarly appealing ex ante, $U_C \in [\underline{U}, \overline{U})$, so the voter re-elects the incumbent if and only if the mainstream media reports $m = x$. A presumption of this is $\underline{U} < \overline{U}$, which, for any given $s > 0$, holds when $l$ is sufficiently low, i.e., subversive types aren't too likely. Second, the effort cost must be sufficiently low, $k \leq q - \frac{1}{2}$, so a high-type incumbent is willing to exert effort to increase her probability of

winning re-election from $\frac{1}{2}$ to $q$. Proposition 1 in the online appendix summarizes equilibria for the baseline model.

## Alternative Media

We now analyze the full model, in which the alternative media reports on whether the incumbent is subversive. To assess the effects of the alternative media, we ask two questions. First, how does it affect the incumbent's policymaking effort? Second, how does it affect electoral selection, both in the sense of re-electing high-type incumbents and in the sense of removing subversive ones?

**Accountability**  The alternative media's reliability plays a key role in determining whether accountability is possible. To see this, we begin with situations in which accountability is possible in the baseline model, $U_C \in [\underline{U}, \overline{U})$ and $k \leq q - \frac{1}{2}$.

  We first consider extreme cases, in which the alternative media is either perfectly reliable ($\phi = 0$) or completely unreliable ($\phi = 1$). If $\phi = 0$, there is an equilibrium with accountability, in which re-election requires not only $m = x$ but also $r = NS$, i.e., the alternative media doesn't allege that the incumbent is subversive. If $\phi = 1$, there is an equilibrium in which the alternative media is ignored and re-election is based solely on the mainstream media message, as in the baseline model.

  If the alternative media's reliability is internal, $\phi \in (0, 1)$, the alternative media's effect is twofold. The first effect concerns the high-type incumbent's effort. If re-election requires $m = x$ and $r = NS$, then increases in $\phi$ demotivate the incumbent, because effort is only rewarded if the alternative media is truthful. To induce effort requires $k \leq \left(q - \frac{1}{2}\right)(1 - \phi)$ or, equivalently,

$$\phi \leq \phi_e := 1 - \frac{k}{q - \frac{1}{2}}.$$

The second effect concerns whether the voter listens to the alternative media. Assuming account-

ability, the voter removes the incumbent when $m = x$ and $r = S$ if

$$U_C \geq U_v := 1 \cdot \Pr(\theta_I = H | m = x, r = S) - s \cdot \Pr(\theta_I = S | m = x, r = S) = \frac{\pi q \phi - sl}{\pi q \phi + (1 - \pi) \frac{1}{2} \phi + l}.$$

Thus the voter only listens to the alternative media if it is sufficiently reliable:

$$\phi \leq \phi_v := \frac{(s + U_C) l}{\pi q (1 - U_C) - (1 - \pi) \frac{1}{2} U_C}.$$

Combining these effects, we see how $\phi$ affects accountability when $U_C \in [\underline{U}, \overline{U})$. As shown in Figure 1, if $\phi_v \leq \phi_e$, then if there is accountability in the baseline, there is also accountability with an alternative media. The voter listens to the alternative media if $\phi \leq \phi_v$ and ignores it otherwise.
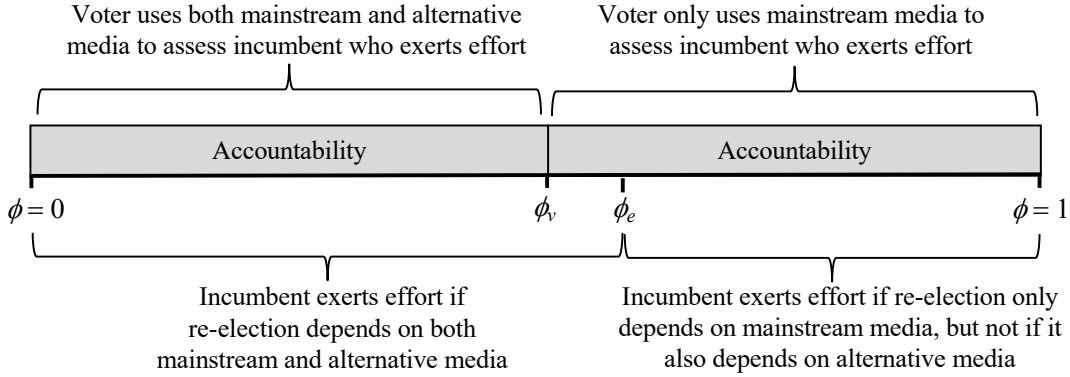


Figure 1: Equilibrium as a function of probability that alternative media is malicious ($\phi$) when effort cost is low ($k < (q - 1/2)(1 - \phi_v)$) and hence $\phi_v < \phi_e$.

If $\phi_v > \phi_e$, the effect on accountability is more nuanced, as shown in Figure 2. If $\phi \leq \phi_e$ or $\phi > \phi_v$, there is accountability and the voter listens to the alternative media in the first case and ignores it in the second. But for intermediate reliability $\phi \in (\phi_e, \phi_v]$, accountability is impossible. This is because the alternative media is sufficiently reliable that the voter listens to it ($\phi \leq \phi_v$), but is sufficiently unreliable that the incumbent's fear of being falsely criticized makes her unwilling to exert effort ($\phi > \phi_e$). A necessary condition for the alternative media to disrupt accountability is $\phi_v > \phi_e$ or, equivalently $k > \left(q - \frac{1}{2}\right)(1 - \phi_v)$, i.e., that policymaking is difficult, in the sense that it is costly for the incumbent to learn the correct policy that serves the voter's interest.
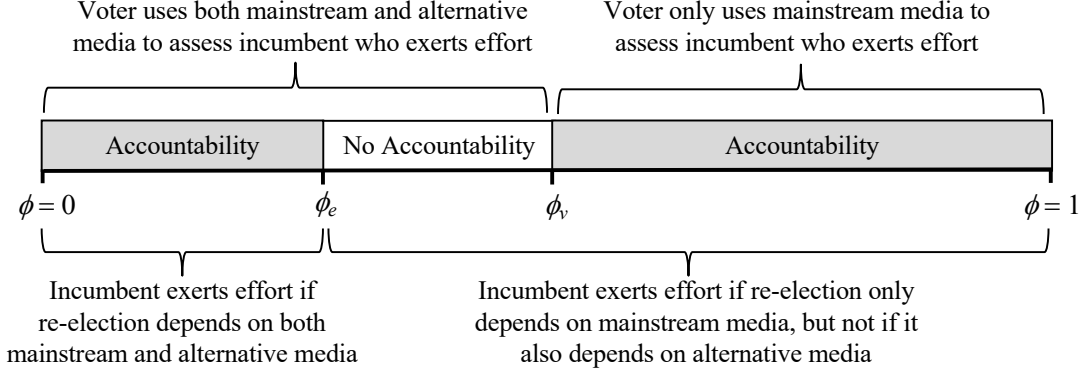
Figure 2: Equilibrium as a function of probability that alternative media is malicious ($\phi$) when effort cost is high ($k > (q - 1/2)(1 - \phi_v)$) and hence $\phi_v > \phi_e$.

In other situations, the alternative media can have positive effects on accountability. If the challenger is highly appealing ($U_C \geq \overline{U}$), accountability is impossible in the baseline model and the voter removes the incumbent even when $m = x$. The alternative media reveals additional information, because $r = NS$ means the incumbent is not subversive. Assuming accountability, the voter's expected utility from re-electing the incumbent after observing $m = x$ and $r = NS$ is:

$$\overline{\overline{U}} := \Pr\left(\theta_I = H | m = x \text{ and } r = NS\right) = \frac{\pi q}{\pi q + (1 - \pi)\frac{1}{2}} > \overline{U}.$$

Thus the alternative media makes accountability possible when the challenger isn't too highly-appealing and false criticism is not so likely as to demotivate effort, $U_C \in [\overline{U}, \overline{\overline{U}})$ and $\phi \leq \phi_e$.

**Selection**   We next analyze how effectively the voter selects competent types and weeds out subversive ones, starting with cases where the challenger is moderately appealing, $U_C \in [\underline{U}, \overline{U})$.

The case $\phi_v \leq \phi_e$ (Figure 1) is most straightforward. For $\phi \leq \phi_v$, there is accountability: the voter listens to both media outlets and weeds out the subversive type. Local increases in $\phi$ worsen selection of the competent type, who is more frequently falsely accused and removed from office. For $\phi > \phi_v$, there is accountability, but the alternative media is ignored, and the subversive type, who induces the mainstream media to report $m = x$, is never weeded out. In this parameter region, local changes in $\phi$ don't affect the selection of either type.

8

The case $\phi_v > \phi_e$ (Figure 2) is more dramatic, because $\phi$ affects incumbent effort. At $\phi = \phi_e$, the equilibrium transitions from one with accountability to one without accountability, so the voter loses the benefit of selecting based on the mainstream media message as $\phi$ crosses this threshold. Then, as $\phi$ increases above $\phi_v$, accountability is restored, but the alternative media is ignored, so selection is based solely on the mainstream media message and the subversive type is never weeded out.

We also note how $\phi$ affects selection based on the mainstream media message when $U_C \notin [\underline{U}, \overline{U})$. With an unappealing or extremely appealing challenger, $U_C < \underline{U}$ or $U_C > \overline{\overline{U}}$, there is no accountability and the voter never selects based on the mainstream media message. With a reasonably highly-appealing challenger, $U_C \in [\overline{U}, \overline{\overline{U}})$, there is accountability if the alternative media is not demotivating, $\phi \leq \phi_e$, in which case the voter uses information from both media outlets.

Finally, we note that absent accountability, the voter benefits from selecting based on the alternative media message if two conditions hold. First, the incumbent must be sufficiently likely to be a high type to win re-election when the voter learns that she is non-subversive ($r = NS$) but learns nothing about her competence. This requires $\pi > U_C$. Moreover, the alternative media must be sufficiently reliable for the voter to remove the incumbent when $r = S$:

$$U_C \geq U_a := 1 \cdot \Pr\left(\theta_I = H | r = S\right) - s \cdot \Pr\left(\theta_I = S | r = S\right) = \frac{\pi\phi - sl}{\phi + l}, \text{ equivalently } \phi \leq \phi_a := \frac{(s + U_C)\,l}{\pi - U_C}.$$

Proposition 2 in the online appendix summarizes equilibria for the model with alternative media.

## Implications

We conclude by discussing several implications of our model.

**Failure to heed warnings about democratic backsliding**  Autocratization is a major global trend, in countries such as Brazil, Hungary, India, Poland, and Turkey. It takes many forms, as incumbents subvert elections, the bureaucracy, judiciary, and other institutions. Scholars have

identified behind-the-scenes control of the media as one of the most common forms of backsliding (Bermeo 2016), and evidence on bribes by President Fujimori of Peru shows that he placed an especially high value on the media (McMillan and Zoido 2004). Alternative media sometimes serve as independent information sources and undermine support for autocrats (Knight and Tribin 2019). But, as noted by Bermeo, they often face a credibility problem, when rulers accuse them of being special interests, representatives of a discredited old order, or tools of foreign powers.

In our model, unless the alternative media is seen as being highly reliable ($\phi < \min\{\phi_e, \phi_v\}$), elections can achieve at most one of two important goals: ensuring accountability for regular incumbents and removing aspiring autocrats (see Figures 1 and 2). When the alternative media is seen as being sufficiently unreliable ($\phi > \phi_v$), voters make electoral decisions based solely on the mainstream media announcement. This means a subversive incumbent can win re-election by inducing the mainstream media to praise her policy choices. Although voters are aware that such praise might just be propaganda, they think the alternative media is probably "crying wolf," so they fail to act on its warnings about the incumbent's subversion, and thus forgo the opportunity to prevent democratic backsliding.

**Diminished trust in mainstream media**   In our model, the voter's belief about the mainstream media's truthfulness decreases, from $1-\sigma$ to $\frac{(1-\sigma)(1-\phi)}{(1-\sigma)(1-\phi)+\sigma}$, when the alternative media alleges that it is acting as an incumbent-controlled propagandist rather than an independent information provider. This is broadly consistent with the recent decline in trust in traditional media outlets at a time when alternative media disparage the mainstream media.

**Fake news and accountability**   Another application of our model is to fake news that alleges that an incumbent is an aspiring autocrat who is in cahoots with the mainstream media. Theories of fake news include Allcott and Gentzkow (2017), Yea (2018), and Taylor (2019). While these models analyze the generation of false claims as well as effects on voter behavior, our model speaks to the accountability effects of fake news. Specifically, the alternative media is most beneficial when it is

genuine and known to be genuine. In the opposite extreme, an alternative media that is known to be fake is ignored and does not disrupt accountability. What is most problematic for accountability is an alternative media that is too genuine to be ignored yet too fake to motivate good policymaking.

Our theory may shed light on possible effects of tech giants' recent measures to battle fake news. In particular, it recommends strong measures adopted by Youtube and Twitter, such as labeling or removing bogus material, rather than Facebook's more lenient approach of only removing material that has been edited via artificial intelligence (Alba and Cogner 2020). In fact, lenient measures can be worse than doing nothing, if they wind up disrupting accountability by inadvertently increasing the credibility of fake news that is not removed from a site. In contrast, strong measures restore voters' confidence in the materials being posted on the platform and reactivate their roles in upholding electoral accountability and selection. A consequence of this is expedited reputation building for truthful alternative media sources, whose intentions could long remain uncertain and indistinguishable from those of a myriad of other sources without the intervention of platform owners.

## Acknowledgements

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Anqi Li is an Assistant Professor of Economics at the Department of Economics, Washington University in St. Louis, St. Louis, MO 63105.

Davin Raiha is a Visiting Assistant Professor of Business, Economics, and Public policy at the Kelly School of Business, Indiana University, Bloomington, IN 47405.

Kenneth W. Shotts is the David S. and Ann M. Barlow Professor of Political Economy at the Stanford Graduate School of Business, Stanford University, Stanford, CA 94305.

# Online Appendix of "Propaganda, Alternative Media, and Accountability in Fragile Democracies"

This online appendix is organized as follows. We first comment on a few modeling assumptions. Next, we state Proposition 1 and 2, which characterize equilibria. We then prove these propositions. Finally, we develop additional results.

## Modeling Assumptions

1. *Propagandist media always non-strategically reports $m = x$.* We assume that the propagandist media always reports that the subversive incumbent chose the correct policy. A natural question is whether a strategic propagandist who seeks to maximize the subversive incumbent's probability of re-election has an incentive to instead report $m \neq x$. Our equilibria remain equilibria even if the propagandist is strategic, because a strategic propagandist cannot help the incumbent by reporting $m \neq x$.

2. *"Good" autocrats.* A possible objection is that some autocrats turn out to be effective policymakers who are well-liked by their subjects. In our model, $-s$ represents voters' *expected* payoff when a subversive autocrat remains in office. This reduced-form assumption allows the subversive type to be good with some probability, as long as the voter's expected payoff from autocrats is negative.

3. *Effort by subversive type.* We assume that the subversive type cannot learn the state of the world. A natural question is whether this matters. It doesn't, because even if the subversive type could exert costly effort to learn what policy serves voters' interests, she would have no incentive to do so, because she can induce the mainstream media to report $m = x$.

4. *Exogenous capture of mainstream media by subversive type.* In our model, we assume that a subversive incumbent controls the mainstream media. The section "Additional Results: Capture and Influence" analyzes a variant in which the incumbent must pay a cost to capture

1

the mainstream media, and shows that our main results continue to hold.

5. *Alternative media not influenced by subversive type.* In our model, we conceptualize the alternative media as being outside of the incumbent's control. The section "Additional Results: Capture and Influence" analyzes a variant in which a subversive incumbent sometimes induces the alternative media to report $r = NS$ rather than $r = S$. In that variant, our main results continue to hold.

6. *Symmetry with respect to $x$.* We assume that low and subversive types treat policies symmetrically, which directly implies that they choose each policy $x \in \{0, 1\}$ with probability $\frac{1}{2}$. In the section "Additional Results: Symmetry" we show that our results on the existence of equilibria with accountability do not depend on this assumption, because for any asymmetric equilibrium with accountability we can construct another, symmetric, equilibrium with accountability, in which the strategies of type $H$ and the voter are unchanged.

7. *Ignoring perverse equilibria.* We focus on equilibria in which incumbents signal competence by choosing correct policies. As in many competence-based models of accountability, our model could admit perverse equilibria in which high types signal competence by choosing wrong policies $(x \neq \omega)$. Given that sort of behavior by high types, a strategic propagandist media would announce $m \neq x$ to make subversive incumbents look skillful. We believe these perverse equilibria are empirically implausible and, hence, ignore them (as standard in the literature).

8. *Tiebreaking assumptions.* For technical simplicity, we assume that a high type incumbent who is indifferent about effort exerts effort and a voter who is indifferent removes the incumbent. This assumption only affects our analysis for knife-edge parameter cases, specifically $k \in \left\{q - \frac{1}{2}, \left(q - \frac{1}{2}\right)(1 - \phi)\right\}$, $U_C \in \left\{\underline{U}, \overline{U}, \overline{\overline{U}}, \pi\right\}$, and $\phi \in \{\phi_e, \phi_v, \phi_a\}$. These are the only parameters for which each of these actors can be indifferent when the other plays a pure strategy.

**Statement of Equilibria**

The following propositions summarize equilibria with accountability whenever it is possible, as well as the voter's re-election rule when accountability is impossible. Throughout our analysis, we focus on parameters such that it is possible to have accountability for some values of $U_C$, i.e., $k \leq q - \frac{1}{2}$, and $l$ is sufficiently small so that $\underline{U} < \overline{U}$.

**Proposition 1.** *With only the mainstream media.*

1. *For $U_C < \underline{U}$, type $H$ doesn't exert effort and the incumbent is always re-elected.*

2. *For $U_C \in [\underline{U}, \overline{U})$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$.*

3. *For $U_C \geq \overline{U}$, type $H$ doesn't exert effort and the incumbent is never re-elected.*

**Proposition 2.** *With an alternative media.*

1. *For $U_C < \underline{U}$, type $H$ doesn't exert effort and two situations can happen:*

   (a) *if $\phi \leq \phi_a := \frac{(s+U_C)l}{\pi - U_C}$, the incumbent is re-elected iff $r = NS$;*

   (b) *if $\phi > \phi_a$, the incumbent is always re-elected.*

2. *For $U_C \in [\underline{U}, \overline{U})$.*

   (a) *If $\phi_v \leq \phi_e$, two situations can happen:*

      i. *if $\phi \leq \phi_v$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$ and $r = NS$;*

      ii. *if $\phi > \phi_v$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$.*

   (b) *If $\phi_v > \phi_e$, three situations can happen:*

      i. *if $\phi \leq \phi_e$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$ and $r = NS$;*

      ii. *if $\phi \in (\phi_e, \phi_v]$, type $H$ doesn't exert effort, and the incumbent is either never re-elected (if $\phi_a < 0$), always re-elected (if $\phi > \phi_a > 0$), or re-elected iff $r = NS$ (if $\phi \leq \phi_a$);*

3

*iii. if $\phi > \phi_v$ type $H$ exerts effort and the incumbent is re-elected iff $m = x$.*

*3. For $U_C \in [\overline{U}, \overline{\overline{U}})$.*

    *(a) If $\phi \leq \phi_e$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$ and $r = NS$.*

    *(b) If $\phi > \phi_e$, type $H$ doesn't exert effort and two situations can happen:*

        *i. if $U_C < \pi$, the incumbent is re-elected iff $r = NS$;*

        *ii. if $U_C \geq \pi$, the incumbent is never re-elected;*

*4. For $U_C \geq \overline{\overline{U}}$, type $H$ doesn't exert effort and the incumbent is never re-elected.*

## Proofs

For any parameter values there exists an equilibrium in which the type $H$ incumbent does not exert effort. We analyze whether it is possible to have an equilibrium with accountability. In the proofs, we use $\mu^H$ and $\mu^S$ to denote the voter's beliefs about the probability that the incumbent is type $H$ or $S$, respectively.

### Proof of Proposition 1

We prove part 2 of the proposition, then parts 1 and 3.

**Part 2.** We first calculate voter beliefs, then turn to voter and incumbent strategies.

Suppose type $H$ exerts effort and chooses $x = \omega$. Recall that the mainstream media reports $m = \omega$ with probability $q$ if the incumbent is type $H$ or $L$ and always reports $m = x$ if the incumbent is type $S$.

For voter beliefs when $m = x$: with probability $(1 - \sigma)\pi$ the incumbent is type $H$ in which case $m = x$ with probability $q$, with probability $(1 - \sigma)(1 - \pi)$ the incumbent is type $L$ in which case $m = x$ with probability $\frac{1}{2}$, and with probability $\sigma$ the incumbent is type $S$ in which case $m = x$

4

always. Using the notation $l := \frac{\sigma}{1-\sigma}$, Bayes' rule implies that

$$\mu^H(m = x) = \frac{(1-\sigma)\,\pi q}{(1-\sigma)\,\pi q + (1-\sigma)\,(1-\pi)\,\frac{1}{2} + \alpha} = \frac{\pi q}{\pi q + (1-\pi)\,\frac{1}{2} + l},$$

$$\text{and} \quad \mu^S(m = x) = \frac{\sigma}{(1-\sigma)\,\pi q + (1-\sigma)\,(1-\pi)\,\frac{1}{2} + \sigma} = \frac{l}{\pi q + (1-\pi)\,\frac{1}{2} + l}.$$

For beliefs when $m \neq x$: with probability $(1-\sigma)\,\pi$ the incumbent is type $H$, in which case $m \neq x$ with probability $(1-q)$; with probability $(1-\sigma)\,(1-\pi)$ the incumbent is type $L$, in which case $m \neq x$ with probability $\frac{1}{2}$; and with probability $\sigma$ the incumbent is type $S$, in which case the media never reports $m \neq x$. Thus

$$\mu^H(m \neq x) = \frac{\pi\,(1-q)}{\pi\,(1-q) + (1-\pi)\,\frac{1}{2}}$$

$$\text{and} \quad \mu^S(m \neq x) = 0.$$

The voter gets utility 1 from type $H$, 0 from type $L$, and $-s$ from type $S$, so his expected utility from re-electing when $m = x$ is

$$1 \cdot \mu^H(m = x) - s \cdot \mu^S(m = x) = \frac{\pi q - sl}{\pi q + (1-\pi)\,\frac{1}{2} + l} = \overline{U},$$

and his expected utility from re-electing when $m \neq x$ is

$$1 \cdot \mu^H(m \neq x) - s \cdot \mu^S(m \neq x) = \frac{\pi\,(1-q)}{\pi\,(1-q) + (1-\pi)\,\frac{1}{2}} = \underline{U}.$$

Thus if the voter's utility from the replacement is $U_C \in [\underline{U}, \overline{U})$, he re-elects the incumbent iff $m = x$.

For type $H$, given the voter's strategy, she is re-elected with probability $\frac{1}{2}$ if she doesn't exert effort and with probability $q$ if she exerts effort, so she exerts effort if the cost satisfies $k \leq q - \frac{1}{2}$.

**Parts 1 and 3.** We first assume accountability and derive a contradiction. If type $H$ exerts effort and chooses $x = \omega$, then for $U_C < \underline{U}$ the voter always re-elects the incumbent, so type $H$ will not exert effort. Likewise, if type $H$ exerts effort and chooses $x = \omega$, then for $U_C \geq \overline{U}$ the voter

always removes the incumbent so type $H$ will not exert effort.

In these situations, given that the incumbent does not exert effort and hence her choice of $x$ does not reveal anything about her type, the voter's expected utility from re-electing the incumbent is the same regardless of whether $m = x$ or $m \neq x$. With probability $(1 - \sigma)\pi$ the incumbent is type $H$, with probability $(1 - \sigma)(1 - \pi)$ she is $L$, and with probability $\sigma$ she is $S$. So the voter's expected utility from re-electing the incumbent is

$$1 \cdot (1 - \sigma)\pi + 0 \cdot (1 - \sigma)(1 - \pi) - s \cdot \sigma = (1 - \sigma)\pi - s\sigma,$$

and the voter re-elects the incumbent iff this expected utility is strictly greater than $U_C$.     □

To set up the proof of Proposition 2, we develop three lemmas to characterize voter beliefs, voter best responses, and incumbent best responses.

**Lemma 1.** *The voter's beliefs depend on type $H$'s effort choice as follows.*

1. *If type $H$ exerts effort, then*

   (a) $\mu^H(m = x, r = NS) = \frac{\pi q}{\pi q + (1 - \pi)\frac{1}{2}}$ *and* $\mu^S(m = x, r = NS) = 0$;

   (b) $\mu^H(m = x, r = S) = \frac{\pi q \phi}{\pi q \phi + (1 - \pi)\frac{1}{2}\phi + l}$ *and* $\mu^S(m = x, r = S) = \frac{l}{\pi q \phi + (1 - \pi)\frac{1}{2}\phi + l}$;

   (c) $\mu^H(m \neq x, r = NS) = \mu^H(m \neq x, r = S) = \frac{\pi(1 - q)}{\pi(1 - q) + (1 - \pi)\frac{1}{2}}$; *and*

   $\mu^S(m \neq x, r = NS) = \mu^S(m \neq x, r = S) = 0.$

2. *If type $H$ does not exert effort, then*

   (a) $\mu^H(m = x, r = NS) = \mu^H(m \neq x, r = NS) = \pi$ *and*

   $\mu^S(m = x, r = NS) = \mu^S(m \neq x, r = NS) = 0$;

   (b) $\mu^H(m = x, r = S) = \mu^H(m \neq x, r = S) = \frac{\pi \phi}{\phi + l}$ *and*

   $\mu^S(m = x, r = S) = \mu^S(m \neq x, r = S) = \frac{l}{\phi + l}.$

6

**Proof** All of the results follow directly from Bayes' rule. We note a few details for different parts of the lemma.

1(a). The alternative media reports $r = NS$ only if the incumbent is not type $S$. Conditional on $r = NS$, with probability $\pi$ the incumbent is type $H$, in which case $m = x = \omega$ with probability $q$. With probability $(1 - \pi)$ the incumbent is type $L$, in which case $m = x$ with probability $\frac{1}{2}$.

1(b). Recall that the alternative media reports $r = S$ for types $H$ and $L$ with probability $\phi$ (the probability that it is malicious) and it always reports $r = S$ when the incumbent is type $S$. So with probability $(1 - \sigma)\pi q \phi$ the incumbent is type $H$ and the media reports are $m = x$ and $r = S$, with probability $(1 - \sigma)(1 - \pi)\frac{1}{2}\phi$ the incumbent is type $L$ and the reports are $m = x$ and $r = S$, and with probability $\sigma$ the incumbent is type $S$ and the reports are $m = x$ and $r = S$. Applying Bayes' rule and using $l = \frac{\sigma}{1-\sigma}$ yields the result.

1(c). When $m \neq x$ the voter knows the incumbent is not type $S$, and straightforwardly updates on the competence of the incumbent.

2(a). The alternative media announcement $r = NS$ eliminates the possibility that the incumbent is type $S$. Because type $H$ does not exert effort, the voter doesn't update about whether the incumbent is type $H$ or $L$ based on $m$.

2(b). The voter doesn't update based on $m$, but does update about the probability that the incumbent is non-subversive, based on $r$. Since $r = S$ with probability $\phi$ if the incumbent is non-subversive and with probability 1 if she is subversive, applying Bayes' Rule gives the desired result. $\square$

**Lemma 2.** *The voter's best responses to type $H$'s effort choice are as follows.*

1. *Suppose type $H$ exerts effort.*

   (a) *When $m = x$ and $r = NS$, the voter re-elects the incumbent iff $U_C < \overline{\overline{U}} = \frac{\pi q}{\pi q + (1-\pi)\frac{1}{2}}$.*

   (b) *When $m = x$ and $r = S$, the voter re-elects the incumbent iff $U_C < U_v = \frac{\pi q \phi - sl}{\pi q \phi + (1-\pi)\frac{1}{2}\phi + l}$ or, equivalently, $\phi > \phi_v = \frac{(s + U_C)l}{\pi q(1 - U_C) - (1-\pi)\frac{1}{2}U_C}$.*

7

(c) When $m \neq x$, regardless of $r$ the voter re-elects the incumbent iff $U_C < \underline{U} = \frac{\pi(1-q)}{\pi(1-q)+(1-\pi)\frac{1}{2}}$.

2. Suppose type $H$ does not exert effort.

   (a) When $r = NS$, regardless of $m$ the voter re-elects the incumbent iff $U_C < \pi$ or, equivalently, $\phi_a = \frac{(s+U_C)l}{\pi-U_C} > 0$.

   (b) When $r = S$, regardless of $m$ the voter re-elects the incumbent iff $U_C < U_a := \frac{\pi\phi - sl}{\phi+l}$ or, equivalently either $U_C \geq \pi$ or, if $U_C < \pi$, $\phi > \phi_a = \frac{(s+U_C)l}{\pi-U_C}$.

**Proof** The voter gets utility 1 from type $H$, 0 from type $L$, and $-s$ from type $S$, so his expected utility from re-electing the incumbent is

$$1 \cdot \mu^H - s\mu^S.$$

The parts of the lemma follow from the corresponding parts of Lemma 1 and algebra. $\qquad\square$

**Lemma 3.** *Type $H$'s best responses to the voter's re-election rule are as follows.*

1. *If the voter re-elects the incumbent iff $m = x$ and $r = NS$, then type $H$ exerts effort iff $k \leq \left(q - \frac{1}{2}\right)(1 - \phi)$ or, equivalently, $\phi \leq \phi_e = 1 - \frac{k}{q-\frac{1}{2}}$.*

2. *If the voter re-elects the incumbent iff $m = x$, then type $H$ exerts effort iff $k \leq q - \frac{1}{2}$.*

3. *If the voter re-elects the incumbent iff $r = NS$, then type $H$ does not exert effort.*

4. *If the voter always re-elects the incumbent or never re-elects the incumbent, then type $H$ does not exert effort.*

**Proof** For Part 1: with effort, type $H$'s expected utility is $q(1 - \phi) - k$, and without effort it is $\frac{1}{2}(1 - \phi)$, which yields the condition. For Part 2: with effort, type $H$'s expected utility is $q - k$, and without effort it is $\frac{1}{2}$, which yields the condition. For Parts 3 and 4, type $H$ does not exert effort because effort is costly and does not affect the probability of re-election. $\qquad\square$

**Proof of Proposition 2**

For each parameter region, we conjecture that the incumbent *behaves accountably* in the following sense: type $H$ exerts effort and chooses $x = \omega$, whereas $L$ and $S$ choose each $x \in \{0, 1\}$ with probability $\frac{1}{2}$. We then characterize the voter's best response. If it's optimal for type $H$ to exert effort given the voter's best response, there is an equilibrium with accountability. Otherwise there isn't, and we characterize the voter's re-election rule when all incumbent types choose each policy with probability $\frac{1}{2}$. Note that $\underline{U}, U_v, U_a < \overline{U} < \overline{\overline{U}}$ and that $\underline{U}, U_a < \pi < \overline{\overline{U}}$.

**Proposition 2.1** Suppose $U_C < \underline{U}$ and the incumbent behaves accountably. By Lemma 2.1(c), the voter re-elects the incumbent if $m \neq x$. Thus type $H$ has no incentive to exert effort, so there cannot be an equilibrium with accountability. Now suppose all incumbent types choose each policy with probability $\frac{1}{2}$. Because $U_C < \underline{U} < \pi$, Lemma 2.2(a) implies that when $r = NS$ the incumbent is re-elected. Lemma 2.2(b) shows that when $r = S$ the voter's best response depends on whether $\phi > \phi_a$, thus completing the proof of Proposition 2.1.

**Proposition 2.2** Suppose $U_C \in [\underline{U}, \overline{U})$ and the incumbent behaves accountably. The voter's best response depends on whether $\phi > \phi_v$. Consider two cases.

Case 1. If $\phi \leq \phi_v$, then $U_v \leq U_C < \overline{\overline{U}}$, so Lemma 2.1 implies that the incumbent is reelected iff $m = x$ and $r = NS$. By Lemma 3.1, it is optimal for type $H$ to exert effort iff $\phi \leq \phi_e$, and combining yields the existence of equilibria with accountability as in Proposition 2.2(a)i and 2.2(b)i. On the other hand, if $\phi \in (\phi_e, \phi_v]$, then Lemma 3.1 implies that it is not optimal for the incumbent to exert effort given the voter's best response. Without incumbent effort, Lemma 2.2 characterizes the voter's behavior, as stated in Proposition 2.2(b)ii.

Case 2. If $\phi > \phi_v$, then $U_C < U_v < \overline{\overline{U}}$, so Lemma 2.1 implies that the incumbent is re-elected iff $m = x$. From Lemma 3.2, it is optimal for type $H$ to exert effort iff $k \leq q - \frac{1}{2}$, which holds true by assumption. Combining yields Proposition 2.2(a)ii and 2.2(b)iii.

**Proposition 2.3**  Suppose $U_C \in [\overline{U}, \overline{\overline{U}})$ and the incumbent behaves accountably. Since $\overline{U} > U_v$

and hence $U_C > U_v$, Lemma 2.1 implies that the incumbent is re-elected iff $m = x$ and $r = NC$.

In addition, by Lemma 3.1, type $H$ exerts effort iff $\phi \leq \phi_e$. Combining yields the condition for the

existence of an equilibrium with accountability as in Proposition 2.3(a).

Without effort by type $H$, first note that $\pi > U_a$. Consider two cases.

Case 1. If $U_C \geq \pi$, then $U_C \geq \pi > U_a$. By Lemma 2, the incumbent is removed regardless of $r$

as in Proposition 2.3(b)ii.

Case 2. If $U_C < \pi$ and $r = NS$, then from Lemma 2.2(a) the incumbent is re-elected. On the

other hand, if $U_C < \pi$ and $r = S$ then the incumbent is removed. To see this, note that $U_C \geq \overline{U}$

by assumption and that $\overline{U} > U_a$. Thus $U_C > U_a$, so from Lemma 2.2(b) the incumbent is removed

when $r = S$. This establishes Proposition 2.3(b)i.

**Proposition 2.4**  Suppose $U_C \geq \overline{\overline{U}}$ and the incumbent behaves accountably. Since $\overline{\overline{U}} > \max\{U_v, \underline{U}, \pi, U_a\}$,

Lemma 2 implies that the incumbent is always removed from office regardless of $r$ and $m$. By Lemma

3.4, type $H$ exerts no effort. □

## Additional Results: Selection

We now summarize a bit of additional analysis of selection, which we left out of the main text for

purposes of brevity.

We first note how $\phi$ affects selection when $U_C \notin [\underline{U}, \overline{U})$. With an unappealing or extremely

appealing challenger, $U_C < \underline{U}$ or $U_C > \overline{\overline{U}}$, there is no accountability and the voter never selects based

on the mainstream media message. With a reasonably highly-appealing challenger, $U_C \in [\overline{U}, \overline{\overline{U}})$,

there is accountability if the alternative media is not demotivating, $\phi \leq \phi_e$, in which case the voter

uses information from both media outlets.

Next, we note that absent accountability, the voter uses the alternative media message for

selection if two conditions hold (from Lemma 2.2). First, the incumbent must be sufficiently likely

to be a high type to win re-election when the voter learns that she is non-subversive ($r = NS$) but

learns nothing about her competence. This requires $\pi > U_C$. Moreover, the alternative media must be sufficiently reliable for the voter to remove the incumbent when $r = S$. This requires $\phi \leq \phi_a$.

## Additional Results: Symmetry

We now show that if there exists an equilibrium with accountability in which types $L$ and $S$ mix with different probabilities than $\frac{1}{2}$, then we can construct another equilibrium with mixing probability $\frac{1}{2}$:

**Proposition 3.** *Fix model primitives. If there exists an equilibrium in which type $H$ exerts effort and chooses $x = \omega$, then there exists an equilibrium in which types $L$ and $S$ choose each $x \in \{0, 1\}$ with probability $\frac{1}{2}$, type $H$ exerts effort and chooses $x = \omega$, and voter's strategy is unchanged compared to the original equilibrium.*

The intuition is as follows. Symmetry implies that types $L$ and $S$ choose each policy with probability $\frac{1}{2}$. If these types were to play asymmetrically, e.g., by choosing $x = 1$ with probability less than $\frac{1}{2}$, this would decrease the voter's expected utility from re-electing the incumbent in information sets where $x = 0$ and increase the voter's expected utility from re-electing the incumbent in information sets where $x = 1$. If such an asymmetric strategy has any effect on the voter's best response (relative to what happens when types $L$ and $S$ play symmetrically), it would be to give types $L$ and $S$ an incentive to play $x = 1$ rather than $x = 0$ in order to increase their chances of winning, thus proving that the asymmetric strategy couldn't be part of an equilibrium. On the other hand, if the asymmetric strategy doesn't affect the voter's best response relative to what happens when types $L$ and $S$ play symmetrically, then it has no effect on type $H$'s incentives and hence accountability.

**Proof**  Let $p_L$ and $p_S$ denote the probabilities that types $L$ and $S$, respectively, choose $x = 1$ in the original equilibrium. Proposition 3 claims that changing these probabilities to $\tilde{p}_L = \tilde{p}_S = \frac{1}{2}$ while holding other players' strategies fixed yields a new equilibrium. This requires showing that types $L$ and $S$ are indifferent between $x \in \{0, 1\}$ and thus can mix. It also requires showing that the voter's

strategy in the original equilibrium remains a best response after the change, and so the incentives for type $H$ to exert effort and choose $x = \omega$ remain unchanged. The proof proceeds in four steps.

**Step 1** Show that in the original equilibrium, the voter must re-elect the incumbent with the same probability in the following two information sets: $m = x = 0$, $r = S$ and $m = x = 1$, $r = S$.

Consider a type $S$ incumbent. For her, the voter always observes $m = x$ and $r = S$. Thus the above two information sets are the only ones that matter for type $S$'s incentive. Suppose, to the contrary, that the voter behaves asymmetrically in these information sets. This means that either

$$EU_V\left(reelect|m = x = 0, r = S\right) \leq U_C < EU_V\left(reelect|m = x = 1, r = S\right),$$

or

$$EU_V\left(reelect|m = x = 1, r = S\right) \leq U_C < EU_V\left(reelect|m = x = 0, r = S\right),$$

where $EU_V\left(reelect \mid \cdot\right)$ denotes the voter's expected utility from re-electing the incumbent in an information set. We focus on the first case. (The analysis of the second is similar and is therefore omitted). In this case, type $S$ is re-elected iff she chooses $x = 1$, so her strict best response is $p_S = 1$. Using this, along with the conjectured $p_L$ and type $H$'s behavior in the conjectured equilibrium, voter beliefs when $x = 0$ are

$$\mu^H(m = x = 0, r = NS) = \mu^H(m = x = 0, r = S) = \frac{\pi q}{\pi q + (1 - \pi)(1 - p_L)},$$

$$\mu^H(m = 1, x = 0, r = NS) = \mu^H(m = 1, x = 0, r = S) = \frac{\pi(1 - q)}{\pi(1 - q) + (1 - \pi)(1 - p_L)},$$

$$\text{and} \quad \mu^S(m, x = 0, r) = 0, \ \forall m, r.$$

Given these beliefs, the voter never re-elects the incumbent when $x = 0$. This follows because

$U_C \geq EU_V\left(reelect | m = x = 0, r = S\right)$ by assumption, and straightforward algebra establishes that

$$
\begin{aligned}
EU_V(reelect | m & = x = 0, r = S) = EU_V\left(reelect | m = x = 0, r = NS\right) \\
& > EU_V\left(reelect | m = 1, x = 0, r = S\right) = EU_V\left(reelect | m = 1, x = 0, r = NS\right).
\end{aligned}
$$

Meanwhile, the voter re-elects the incumbent when $x = m = 1$ and $r = S$ by assumption. Combining shows that all incumbent types strictly prefer to choose $x = 1$ rather than to choose $x = 0$, so in particular type $H$ does not exert effort, thus reaching a contradiction.

**Step 2** Show that if $\tilde{p}_L = \tilde{p}_S = \frac{1}{2}$ and type $H$ exerts effort and chooses $x = \omega$, then the voter's best responses in the two information sets considered in Step 1 ($m = x = 0$, $r = S$ and $m = x = 1$, $r = S$) are the same as in the original equilibrium.

There are two ways that the voter can behave symmetrically: re-electing the incumbent in both information sets, or removing her in both information sets. We focus on the first case, i.e., $U_C < EU_V\left(reelect | m = x = 0, r = S\right), EU_V\left(reelect | m = x = 1, r = S\right)$, since the proof of the second case is similar.

In the original equilibrium the voter's beliefs in the above information sets are

$$
\begin{aligned}
\mu^H(m = x = 0, r = S) &= \frac{\pi q \phi}{\pi q \phi + (1 - \pi)(1 - p_L)\phi + 2l(1 - p_S)}, \\
\mu^S(m = x = 0, r = S) &= \frac{2l(1 - p_S)}{\pi q \phi + (1 - \pi)(1 - p_L)\phi + 2l(1 - p_S)}, \\
\mu^H(m = x = 1, r = S) &= \frac{\pi q \phi}{\pi q \phi + (1 - \pi)p_L \phi + 2l p_S}, \quad \text{and} \\
\mu^S(m = x = 1, r = S) &= \frac{2l p_S}{\pi q \phi + (1 - \pi)p_L \phi + 2l p_S}.
\end{aligned}
$$

Moreover, we have

$$EU_V \left(reelect|m = x = 0, r = S\right) = \frac{\pi q\phi - 2sl\left(1 - p_S\right)}{\pi q\phi + (1 - \pi)\left(1 - p_L\right)\phi + 2l\left(1 - p_S\right)} > U_C$$

$$\text{and } EU_V \left(reelect|m = x = 1, r = S\right) = \frac{\pi q\phi - 2slp_S}{\pi q\phi + (1 - \pi)p_L\phi + 2lp_S} > U_C$$

by assumption, and rearranging yields

$$\pi q\phi \left(1 - U_C\right) > \left(1 - p_L\right) U_C \left(1 - \pi\right)\phi + 2\left(1 - p_S\right) l\left(s + U_C\right), \tag{1}$$

and

$$\pi q\phi \left(1 - U_C\right) > p_L U_C \left(1 - \pi\right)\phi + 2p_S l\left(s + U_C\right). \tag{2}$$

Equations 1 and 2 imply that for any $z \in [0, 1]$,

$$\pi q\phi \left(1 - U_C\right) > z\left[\left(1 - p_L\right) U_C \left(1 - \pi\right)\phi + 2\left(1 - p_S\right) l\left(s + U_C\right)\right]$$
$$+ (1 - z)\left[p_L U_C \left(1 - \pi\right)\phi + 2p_S l\left(s + U_C\right)\right].$$

In particular, for $z = \frac{1}{2}$,

$$\pi q\phi \left(1 - U_C\right) > \frac{1}{2} U_C \left(1 - \pi\right)\phi + \frac{1}{2} 2l\left(s + U_C\right),$$

which, using $\tilde{p}_L = \tilde{p}_S = \frac{1}{2}$, can be written as

$$\frac{\pi q\phi - 2sl\tilde{p}_S}{\pi q\phi + (1 - \pi)\tilde{p}_L\phi + 2l\tilde{p}_S} > U_C. \tag{3}$$

Now, notice that after we change $p_L$ and $p_S$ to $\tilde{p}_L = \tilde{p}_S = \frac{1}{2}$ while holding type $H$'s strategy fixed,

14

the voter's expected utilities from re-electing the incumbent, denoted by $\widetilde{EU}_V \left( reelect \mid \cdot \right)$, become

$$\widetilde{EU}_V \left( reelect \mid m = x = 1, r = S \right) = \widetilde{EU}_V \left( reelect \mid m = x = 0, r = S \right)$$

$$= \frac{\pi q \phi - 2 s l \tilde{p}_S}{\pi q \phi + \left( 1 - \pi \right) \tilde{p}_L \phi + 2 l \tilde{p}_S}$$

and are strictly greater than $U_C$ by Equation 3. Thus in the above information sets, the voter always re-elects the incumbent as in the original equilibrium, thus completing the proof of Step 2.

From Steps 1 and 2, we conclude that type $S$ is indifferent between $x \in \{0, 1\}$ given the voter's best response to $\tilde{p}_L = \tilde{p}_S = \frac{1}{2}$ and type $H$ exerting effort and choosing $x = \omega$. That is, type $S$ can mix after the proposed change.

For the rest of the analysis we focus on type $L$. We begin with three observations:

1. For type $L$, whether $m = x$ is independent of whether $r = S$, and neither depends on her choice of $x$. Thus when characterizing $L$'s best responses we can focus exclusively on whether she is re-elected in the various information sets without worrying about the probability that each information set is reached.

2. Because the probabilities of reaching the two information sets $m = x = 0$, $r = S$ and $m = x = 1$, $r = S$ do not depend on $x$ for type $L$, and Step 2 established that the voter treats the policies symmetrically in these two information sets (either re-electing the incumbent in both or removing her in both), we can ignore these two information sets when analyzing type $L$'s incentives.

3. The remainder of the proof focuses on the other six information sets: $m = x = 0, r = NS$; $m = x = 1, r = NS$; $m = 1, x = 0, r = NS$; $m = 0, x = 1, r = NS$; $m = 1, x = 0, r = S$; and $m = 0, x = 1, r = S$. Since these information sets are never reached for a type $S$ incumbent (for whom $m = x$ and $r = S$ always), there the voter knows that the incumbent is not type $S$, so his beliefs about the incumbent only depend on $p_L$, not $p_S$.

With these observations in place, we continue the analysis of type $L$.

**Step 3**  Show that in the original equilibrium, the $\mu^H$'s in the above six information sets are ordered, such that the voter has a higher opinion of an incumbent who chooses the policy that is less likely to be chosen by type $L$.

These beliefs are

$$\mu^H(m = x = 0, r = NS) = \frac{\pi q}{\pi q + (1 - \pi)(1 - p_L)},$$

$$\mu^H(m = x = 1, r = NS) = \frac{\pi q}{\pi q + (1 - \pi) p_L},$$

$$\mu^H(m = 1, x = 0, r = NS) = \mu^H(m = 1, x = 0, r = S) = \frac{\pi(1 - q)}{\pi(1 - q) + (1 - \pi)(1 - p_L)},$$

$$\text{and } \mu^H(m = 0, x = 1, r = NS) = \mu^H(m = 0, x = 1, r = S) = \frac{\pi(1 - q)}{\pi(1 - q) + (1 - \pi) p_L},$$

and straightforward algebra establishes that

$$\mu^H(m = x = 0, r = NS) < \mu^H(m = x = 1, r = NS)$$

$$\text{iff} \quad \mu^H(m = 1, x = 0, r = NS) < \mu^H(m = 0, x = 1, r = NS)$$

$$\text{iff} \quad \mu^H(m = 1, x = 0, r = S) < \mu^H(m = 0, x = 1, r = S)$$

$$\text{iff} \quad p_L < \frac{1}{2}. \tag{4}$$

Also note that the inequalities are reversed if $p_L > \frac{1}{2}$ and they become equalities if $p_L = \frac{1}{2}$. Finally, because the voter knows the incumbent is not type $S$, the above beliefs are equal to his expected utilities from re-electing the incumbent. That is, $EU_V(reelect \mid m, x, r) = \mu^H(m, x, r)$ in the above six information sets.

**Step 4**  We now complete the proof, analyzing two cases.

**Case 1.** $p_L = \frac{1}{2}$. After changing $p_L$ to $\tilde{p}_L = \frac{1}{2}$, the voter's best responses remain unchanged in

the above six information sets, and because beliefs are equal in each line of Equation 4 he treats the policies symmetrically. Combined with Step 2 (which established that the voter treats the policies symmetrically in the other two information sets where $m = x$ and $r = S$), we obtain that type $L$ is indifferent between the policies and thus can mix after the proposed change. Furthermore, since the voter's strategy in the original equilibrium remains a best response, type $H$'s incentives remain unchanged as well after the proposed change. Taken together, there exists an equilibrium with accountability in which $\tilde{p}_A = \tilde{p}_L = \frac{1}{2}$.

**Case 2.** $p_L \neq \frac{1}{2}$. By symmetry, consider the case of $p_L < \frac{1}{2}$, in which the voter's belief given $x = 0$ is lower than its counterpart given $x = 1$ in each line of Equation 4.

We first prove that the voter treats the policies symmetrically. If the contrary were true, then

$$U_C \in [\mu^H(m = x = 0, r = NS), \mu^H(m = x = 1, r = NS)), \text{ or}$$

$$U_C \in [\mu^H(m = 1, x = 0, r = NS), \mu^H(m = 0, x = 1, r = NS)), \text{ or}$$

$$U_C \in [\mu^H(m = 1, x = 0, r = S), \mu^H(m = 0, x = 1, r = S)),$$

implying that the incumbent is re-elected for $x = 1$ but not for $x = 0$ in at least one of the following three information sets: $m = x$ and $r = NS$, $m \neq x$ and $r = NS$, or $m \neq x$ and $r = S$. However $p_L < \frac{1}{2}$ implies that in the other two information sets, the voter's belief is strictly higher when $x = 1$ than when $x = 0$, so in those information sets he must be weakly more likely to re-elect the incumbent when $x = 1$. Thus type $L$'s best response is to choose $x = 1$ always, which contradicts the assumption that $p_L < \frac{1}{2}$.

Thus the voter must treat the policies symmetrically in the original equilibrium, which requires

that

$$U_C \notin [\mu^H(m = x = 0, r = NS), \mu^H(m = x = 1, r = NS)),$$

$$U_C \notin [\mu^H(m = 1, x = 0, r = NS), \mu^H(m = 0, x = 1, r = NS)), \text{ and}$$

$$U_C \notin [\mu^H(m = 1, x = 0, r = S), \mu^H(m = 0, x = 1, r = S)).$$

The voter continues to treat the policies symmetrically after we replace $p_L$ with $\tilde{p}_L = \frac{1}{2}$ because, using ˜s to denote the new beliefs, Equation 4 implies that the new beliefs are sandwiched by the old ones:

$$\mu^H(m = x = 0, r = NS) < \tilde{\mu}^H(m = x = 0, r = NS)$$

$$= \tilde{\mu}^H(m = x = 1, r = NS) < \mu^H(m = x = 1, r = NS),$$

$$\mu^H(m = 1, x = 0, r = NS) < \tilde{\mu}^H(m = 1, x = 0, r = NS)$$

$$= \tilde{\mu}^H(m = 0, x = 1, r = NS) < \mu^H(m = 0, x = 1, r = NS), \text{ and}$$

$$\mu^H(m = 1, x = 0, r = S) < \tilde{\mu}^H(m = 1, x = 0, r = S)$$

$$= \tilde{\mu}^H(m = 0, x = 1, r = S) < \mu^H(m = 0, x = 1, r = S).$$

Combined with Step 2 (which established that the voter treats the policies symmetrically in the two information sets where $m = x$ and $r = S$), we obtain that type $L$ is indifferent between the policies and thus can mix after the proposed change. The remainder of the proof is the same as in Case 1 and is therefore omitted. □

## Additional Results: Capture and Influence

We now show that our main results continue to hold qualitatively in an extension in which the subversive type decides whether to capture the mainstream media and also has some ability to influence the alternative media.

To endogenize capture of the mainstream media, we extend the model as follows. Rather than assuming that the subversive type automatically controls the mainstream media, we add an initial stage of the game, in which the incumbent can capture the mainstream media at cost $c_{\theta_I}$. The voter does not observe whether the incumbent captures the mainstream media. If captured, the mainstream media always reports $m = x$. Absent capture, it reports via the same technology as in the main model, issuing a report $m \in \{0, 1\}$ that matches the true state $\omega$ with probability $q \in \left(\frac{1}{2}, 1\right)$. We assume $c_H, c_L > \frac{1}{2}$, which immediately implies that for non-subversive incumbents the cost is higher than any possible electoral benefit from capture. Whether the subversive type is willing to capture the mainstream media depends on voter behavior as well as her cost of capture, $c_S$. When indifferent between capturing and not capturing, she captures.

To model the subversive type's influence over the alternative media, we alter the model as follows. Rather than assuming that the alternative media is outside of the incumbent's control, we assume that the subversive type partially controls the alternative media, so rather than always reporting $r = S$ it reports $r = NS$ with probability $\tau \in [0, 1-\phi)$. We interpret the subversive type's influence as being achieved via a combination of censorship that blocks $r = S$ messages along with fabrication of fake $r = NS$ messages by alternative media outlets are aligned with the subversive incumbent. The assumption that $\tau < 1 - \phi$ implies $1 - \tau > \phi$ so that $\Pr\left(r = S | \theta_I = S\right) > \Pr\left(r = S | \theta_I = H \text{ or } L\right)$. This is intuitive—it means that a report $r = S$ increases the voter's belief about the probability that the incumbent is subversive.

To analyze capture and influence, we augment the model in two steps. First, in Proposition 4 we add influence over the alternative media to the model in our main text and show that a variant of Proposition 2 continues to hold, just with different cutpoints. Then, in Propositions 5 and 6 we add endogenous capture as well, and establish sufficient conditions for the cost of capture such that the equilibria in Proposition 1 (without the alternative media) and Proposition 4 (with the alternative media) continue to be equilibria.

Throughout the analysis we focus on the case of moderately-appealing challengers, i.e., $U_C \in$

$[\underline{U}, \overline{U})$, which is the key parameter region for the main results of our paper.

**Influence over alternative media**   The key change caused by the type $S$ incumbent's influence over the alternative media—i.e., that it induces it to report $r = NS$ with probability $\tau$—is that the voter's expected utility from re-electing the incumbent decreases in information sets where $r = NS$ and increases in information sets where $r = S$, relative to the corresponding beliefs in our main model. This, in turn, affects the cutpoints for whether the voter re-elects the incumbent. We write these cutpoints as a function of $\tau$.

We first consider the cutpoint for whether the voter removes the incumbent when $m = x$ and $r = S$. Assuming accountability, the voter does so iff

$$U_C \geq U_v(\tau) := 1 \cdot \Pr(\theta_I = H | m = x, r = S) - s \cdot \Pr(\theta_I = S | m = x, r = S) = \frac{\pi q \phi - s(1 - \tau)l}{\pi q \phi + (1 - \pi)\frac{1}{2}\phi + (1 - \tau)l}.$$

Thus we modify the threshold reliability at which the voter only listens to the alternative media, as follows:

$$\phi \leq \phi_v(\tau) := \frac{(s + U_C)(1 - \tau)l}{\pi q(1 - U_C) - (1 - \pi)\frac{1}{2}U_C}.$$

We next consider the cutpoint for whether, absent accountability and with type $H$ not exerting effort, the voter removes the incumbent when $r = S$:

$$U_C \geq U_a(\tau) := 1 \cdot \Pr(\theta_I = H | r = S) - s \cdot \Pr(\theta_I = S | r = S) = \frac{\pi \phi - s(1 - \tau)l}{\phi + (1 - \tau)l}.$$

This is equivalent to either $\pi \leq U_C$, or $\pi > U_C$ and the alternative media is sufficiently reliable,

$$\phi \leq \phi_a(\tau) := \frac{(s + U_C)(1 - \tau)l}{\pi - U_C}.$$

20

Also, absent accountability the voter re-elects the incumbent when $r = NS$ iff

$$U_C < U_{a2}(\tau) := \frac{\pi(1-\phi) - s\tau l}{(1-\phi) + \tau l}$$

equivalently $\quad \pi > U_C$ and $\phi < \phi_{a2}(\tau) := \frac{(\pi - U_C) - \tau l(U_C + s)}{\pi - U_C}.$

Finally, note that cutpoint $\phi_e$ for effort by a type $H$ incumbent is unaffected by the fact that the alternative media is captured.

Using the cutpoints $\phi_v(\tau)$, $\phi_a(\tau)$ and $\phi_{a2}(\tau)$, we obtain the following result, which closely parallels Proposition 2.2.

**Proposition 4.** *Suppose a type $S$ incumbent captures the mainstream media and influences the alternative media, and $U_C \in [\underline{U}, \overline{U})$.*

(a) *If $\phi_v(\tau) \leq \phi_e$, two situations can happen:*

  i. *if $\phi \leq \phi_v(\tau)$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$ and $r = NS$;*

  ii. *if $\phi > \phi_v(\tau)$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$.*

(b) *$\phi_v(\tau) > \phi_e$, three situations can happen:*

  i. *if $\phi \leq \phi_e$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$ and $r = NS$;*

  ii. *if $\phi \in (\phi_e, \phi_v(\tau)]$, type $H$ doesn't exert effort and the incumbent is either never re-elected (if $\phi_a(\tau) < 0$ or $\phi_a(\tau) > 0$ and $\phi \geq \phi_{a2}(\tau)$), always re-elected (if $\phi_a(\tau) > 0$ and $\phi > \phi_a(\tau)$), or re-elected iff $r = NS$ (if $\phi_a(\tau) > 0$ and $\phi \leq \min\{\phi_a(\tau), \phi_{a2}(\tau)\}$);*

  iii. *if $\phi > \phi_v(\tau)$, type $H$ exerts effort and the incumbent is re-elected iff $m = x$.*

**Proof**   The argument closely parallels the proof of Proposition 2, so we don't provide every detail, but rather note how specific beliefs and cutpoints need to be adjusted to take into account type $S$'s influence on the alternative media.

Paralleling Lemma 1, the voter's beliefs depend on type $H$'s effort choice. If type $H$ exerts effort, the voter's beliefs in information sets where $m \neq x$ are the same as in Lemma 1, because the voter knows the incumbent is not type $S$. Beliefs in information sets where $m = x$ are different, as follows:

$$\mu^H(m = x, r = NS) = \frac{\pi q (1 - \phi)}{\pi q (1 - \phi) + (1 - \pi) \frac{1}{2} (1 - \phi) + \tau l}$$

$$\mu^S(m = x, r = NS) = \frac{\tau l}{\pi q (1 - \phi) + (1 - \pi) \frac{1}{2} (1 - \phi) + \tau l}$$

$$\mu^H(m = x, r = S) = \frac{\pi q \phi}{\pi q \phi + (1 - \pi) \frac{1}{2} \phi + (1 - \tau) l}$$

$$\text{and } \mu^S(m = x, r = S) = \frac{(1 - \tau) l}{\pi q \phi + (1 - \pi) \frac{1}{2} \phi + (1 - \tau) l}.$$

If type $H$ does not exert effort, all voter beliefs must be adjusted from those in Lemma 1, to account for type $S$'s influence over the alternative media, as follows:

$$\mu^H(m = x, r = NS) = \mu^H(m \neq x, r = NS) = \frac{\pi (1 - \phi)}{(1 - \phi) + \tau l}$$

$$\mu^S(m = x, r = NS) = \mu^S(m \neq x, r = NS) = \frac{\tau l}{(1 - \phi) + \tau l}$$

$$\mu^H(m = x, r = S) = \mu^H(m \neq x, r = S) = \frac{\pi \phi}{\phi + (1 - \tau) l}$$

$$\text{and } \mu^S(m = x, r = S) = \mu^S(m \neq x, r = S) = \frac{(1 - \tau) l}{\phi + (1 - \tau) l}.$$

With these beliefs, the voter's best responses take the same form as in Lemma 2. For all parts of the lemma except part 2(a), this simply involves substituting in cutpoints parametrized in terms of $\tau$. Along with $U_v(\tau)$, $\phi_v(\tau)$, $U_a(\tau)$, $\phi_a(\tau)$, and $\phi_{a2}(\tau)$ defined above our statement of Proposition 4, let $\overline{\overline{U}}(\tau) := \frac{\pi q(1-\phi) - s\tau l}{\pi q(1-\phi) + (1-\pi)\frac{1}{2}(1-\phi) + \tau l}$ denote the voter's expected utility from re-electing the incumbent after observing $m = x$ and $r = NS$. We also state $\underline{U}(\tau)$ and $\overline{U}(\tau)$ in terms of $\tau$, $\underline{U}(\tau) := \frac{\pi(1-q)}{\pi(1-q) + (1-\pi)\frac{1}{2}}$ and $\overline{U}(\tau) := \frac{\pi q - sl}{\pi q + (1-\pi)\frac{1}{2} + l}$, though note that these thresholds are independent of $\tau$.

The condition for the voter's best response when type $H$ does not exert effort and $r = NS$ is

slightly more complicated than in Lemma 2.2(a). Regardless of $m$ the voter re-elects the incumbent iff $U_C < U_{a2}(\tau)$, or, equivalently $\pi > U_C$ and $\phi < \phi_{a2}(\tau)$.

It is straightforward to show that $U_{a2}(\tau) > U_a(\tau)$. Focusing on cases where $\pi > U_C$, we also see that $U_C < U_{a2}(\tau)$ iff $\phi < \phi_{a2}(\tau)$ and $U_C \geq U_a(\tau)$ iff $\phi \leq \phi_a(\tau)$. Thus for $\pi > U_C$ $\phi \geq \phi_{a2}(\tau)$ implies $\phi < \phi_a(\tau)$, so Proposition 4(b)ii covers all relevant cases for the relationship of $\phi$ to these cutpoints.

Incumbent best responses are the same as the ones characterized in Lemma 3.

Straightforward algebra establishes that $\underline{U}(\tau), U_v(\tau), U_a(\tau) < \overline{U}(\tau) < \overline{\overline{U}}(\tau)$.

With these results in hand, the rest of the argument follows along the same lines as the proof for Proposition 2.2. $\qquad \square$

**Endogenous capture** We now add to the model an initial stage, in which type $S$ decides whether to pay cost $c_S$ to capture the mainstream media. A captured media reports $m = x$ whereas an uncaptured one reports $m \in \{0,1\}$ that matches the true state $\omega$ with probability $q \in \left(\frac{1}{2}, 1\right)$. We provide sufficient conditions for the cost of capture such that the equilibria in Proposition 1 (without the alternative media) and Proposition 4 (with the alternative media) continue to be equilibria.

We first analyze endogenous capture without an alternative media.

**Proposition 5.** *Without an alternative media, if $U_C \in [\underline{U}, \overline{U})$ and $c_S \leq \frac{1}{2}$, then there exists an equilibrium with accountability, in which type $S$ captures the mainstream media, type $H$ exerts effort, and the incumbent is re-elected iff $m = x$, as in Proposition 1.2.*

**Proof** Given that the incumbent is re-elected iff $m = x$, if type $S$ does not capture the mainstream media, she wins re-election with probability $\frac{1}{2}$. With capture, she can ensure that $m = x$ and thus wins re-election with probability 1. Thus, capture is a best response for type $S$ if $c_S \leq \frac{1}{2}$. The rest of the argument follows the proof for Proposition 1.2. $\qquad \square$

Next we add endogenous capture to the model in which type $S$ influences the alternative media. We establish conditions on the cost of capture under which the equilibria in Proposition 4 continue

to be equilibria when capture is endogenous.

**Proposition 6.** *Suppose type $S$ influences the alternative media and $U_C \in [\underline{U}, \overline{U})$.*

1. *If $\phi \leq \min\{\phi_e, \phi_v(\tau)\}$ and $c_S \leq \frac{\tau}{2}$, then there exists an equilibrium in which type $S$ captures the mainstream media, type $H$ exerts effort, and the incumbent is re-elected iff $m = x$ and $r = NS$, as in Proposition 4(a)i and (b)i.*

2. *If $\phi > \phi_v(\tau)$ and $c_S \leq \frac{1}{2}$, then there exists an equilibrium in which type $S$ captures the mainstream media, type $H$ exerts effort, and the incumbent is re-elected iff $m = x$, as in Proposition 4(a)ii and (b)iii.*

3. *If $\phi \in (\phi_e, \phi_v(\tau)]$, then there does not exist an equilibrium in which type $S$ captures the mainstream media. There exists an equilibrium in which type $H$ doesn't exert effort and the voter uses the same re-election rule as in Proposition 4(b)ii.*

**Proof  Part 1.** Given that the incumbent is re-elected iff $m = x$ and $r = NS$, if type $S$ does not capture the mainstream media she wins re-election with probability $\frac{1}{2}\tau$. With capture, she can ensure that $m = x$ and thus wins re-election with probability $\tau$. Thus, capture is a best response for type $S$ if $c_S \leq \frac{\tau}{2}$.

   **Part 2.** Given that the incumbent is re-elected iff $m = x$, if type $S$ does not capture the mainstream media she wins re-election with probability $\frac{1}{2}$. With capture, she can ensure that $m = x$ and thus wins re-election with probability 1. Thus, capture is a best response for type $S$ if $c_S \leq \frac{1}{2}$.

   **Part 3.** There can't be an equilibrium with accountability in which type $S$ captures the mainstream media, because $\phi > \phi_e$ implies type $H$ is not willing to exert effort when re-election requires $m = x$ and $r = NS$. With the behavior for type $H$ and the voter in Proposition 4(b)ii, type $S$ has no incentive to capture the mainstream media. Absent accountability, the mainstream media is ignored by the voter, who selects only based on the alternative media message. Despite the fact

that type $S$ doesn't capture (which is assumed in Proposition 4), the voter behavior described in Proposition 4(b)ii remains a best response. □

Propositions 5 and 6 show that if subversive incumbents influence the alternative media and endogenously decide whether to capture the mainstream media, then as long as the cost of capture is not too high, there are equilibria that are qualitatively similar to the ones we analyze in the main text. It also would be possible to endogenize influence over the alternative media. While we do not do this, it is easy to see that subversive incumbents would be willing to pay a cost to obtain influence for parameter regions where $r = NS$ helps her win re-election, as long as that cost is not too high.